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* Modeling Standard

(+) indicates additional mathematics that students should learn to prepare for advanced courses
Introduction

The College and Career Ready State Standards for Mathematics (CCSSM) at the high school level specify the mathematics that all students should study in order to be college and career ready. The high school standards are listed in conceptual categories (number and quantity, algebra, functions, geometry, modeling, and probability and statistics). Consideration of how to organize the CCSSM high school standards into courses that provides a strong foundation for post secondary success was needed. To answer this charge, a group of experts, including state mathematics experts, teachers, mathematics faculty from two and four year institutions, mathematics teacher educators, and workforce representatives, were convened to develop Model Course Pathways in high school based on College and Career Ready State Standards for Mathematics (CCSSM). The model pathways can be found in Appendix A of the College and Career Ready State Standards for Mathematics.

After a review of these pathways, the superintendents of Maryland’s LEA’s voted to adopt the pathway reflected in this framework document which is referred to as the “Traditional Pathway”. The “Traditional Pathway” consists of two algebra courses and a geometry course. The standards from the Statistics Conceptual Category are divided between Algebra I and Algebra II.

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How to Read the Maryland College and Career Ready Curriculum Framework for Algebra I

This framework document provides an overview of the standards that are grouped together to form the units of study for Algebra I. The standards within each unit are grouped by conceptual category and are in the same order as they appear in the College and Career Ready State Standards for Mathematics. This document is not intended to convey the exact order in which the standards within a unit will be taught nor the length of time to devote to the study of the unit.

The framework contains the following:

- **Units** are intended to convey coherent groupings of content.
- **Clusters** are groups of related standards. A description of each cluster appears in the left column.
- **Cluster Notes** are instructional statements which relate to an entire cluster of standards. These notes are placed in the center column above all of the standards in the cluster.
- **Essential Skills and Knowledge** statements provide language to help teachers develop common understandings and valuable insights into what a student must understand and be able to do to demonstrate proficiency with each standard. Maryland mathematics educators thoroughly reviewed the standards and, as needed, provided statements to help teachers comprehend the full intent of each standard. The wording of some standards is so clear, however, that only partial support or no additional support seems necessary.
- **Standards** define what students should understand and be able to do.
- **Notes** are instructional notes that pertain to just one standard. They are placed in the center column immediately under the standard to which they apply. The notes provide constraints, extensions and connections that are important to the development of the standard.
- **Standards for Mathematical Practice** are listed in the right column.
- ★ Denotes that the standard is a Modeling Standard. Modeling standards are woven throughout each conceptual category.
- (+) indicates additional mathematics that students should learn to prepare for advanced courses.

### Formatting Notes

- **Red Bold** - information added by Maryland educators
- **Blue bold** – words/phrases that are linked to clarifications
- **Black bold underline** - words within repeated standards that indicate the portion of the statement that is emphasized at this point in the curriculum or words that draw attention to an area of focus
- **Black bold** - Cluster Notes-notes that pertain to all of the standards within the cluster
- **Green bold** – standard codes from other courses that are referenced and are hot linked to a full description

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Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in

★ Modeling Standard

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Modeling Standard

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4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection...
of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the $14$ as $2 \times 7$ and the $9$ as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as $5$ minus a positive number times a square and use that to realize that its value cannot be more than $5$ for any real numbers $x$ and $y$.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing $25$ by $11$ that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope $3$, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

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Modeling Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

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The basic modeling cycle is summarized in the diagram.

It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO₂ over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

**Modeling Standards** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol ★.

**(+)** Standards The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.
### Grades K-8

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<th>Domain Name</th>
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<tbody>
<tr>
<td>CC</td>
<td>Counting &amp; Cardinality</td>
<td>K</td>
</tr>
<tr>
<td>EE</td>
<td>Expressions &amp; Equations</td>
<td>6, 7, 8</td>
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<tr>
<td>F</td>
<td>Functions</td>
<td>8</td>
</tr>
<tr>
<td>G</td>
<td>Geometry</td>
<td>K, 1, 2, 3, 4, 5, 6, 7, 8</td>
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<tr>
<td>MD</td>
<td>Measurement &amp; Data</td>
<td>K, 1, 2, 3, 4, 5</td>
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<tr>
<td>NBT</td>
<td>Number &amp; Operations (Base Ten)</td>
<td>K, 1, 2, 3, 4, 5</td>
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<tr>
<td>NF</td>
<td>Number &amp; Operations (Fractions)</td>
<td>3, 4, 5</td>
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<tr>
<td>NS</td>
<td>Number System</td>
<td>6, 7, 8</td>
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<tr>
<td>OA</td>
<td>Operations &amp; Algebraic Thinking</td>
<td>K, 1, 2, 3, 4, 5</td>
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<tr>
<td>RP</td>
<td>Ratios &amp; Proportional Relationship</td>
<td>6, 7</td>
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<tr>
<td>SP</td>
<td>Statistics &amp; Probability</td>
<td>6, 7, 8</td>
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### Modeling

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### High School

#### Algebra (A)

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<tr>
<td>A-APR</td>
<td>Arithmetic with Polynomial &amp; Rational Expressions</td>
<td>8–12</td>
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<tr>
<td>A-CED</td>
<td>Creating Equations</td>
<td>8–12</td>
</tr>
<tr>
<td>A-REI</td>
<td>Reasoning with Equations &amp; Inequalities</td>
<td>8–12</td>
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<tr>
<td>A-SSE</td>
<td>Seeing Structure in Expressions</td>
<td>8–12</td>
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#### Functions (F)

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<tbody>
<tr>
<td>F-BF</td>
<td>Building Functions</td>
<td>8–12</td>
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<td>F-IF</td>
<td>Interpreting Functions</td>
<td>8–12</td>
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<tr>
<td>F-LE</td>
<td>Linear, Quadratic &amp; Exponential Models</td>
<td>8–12</td>
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<td>F-TF</td>
<td>Trigonometric Functions</td>
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### Geometry (G)

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<td>G-C</td>
<td>Circles</td>
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<td>G-CO</td>
<td>Congruence</td>
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<td>G-GMD</td>
<td>Geometric Measurement &amp; Dimension</td>
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<td>G-MG</td>
<td>Modeling with Geometry</td>
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<td>G-GPE</td>
<td>Expressing Geometric Properties with Equations</td>
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<td>G-SRT</td>
<td>Similarity, Right Triangles &amp; Trigonometry</td>
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### Number & Quantity (N)

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<td>N-CN</td>
<td>Complex Number System</td>
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<td>N-Q</td>
<td>Quantities</td>
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<tr>
<td>N-RN</td>
<td>Real Number System</td>
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<tr>
<td>N-VM</td>
<td>Vector &amp; Matrix Quantities</td>
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### Statistics (S)

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<td>S-ID</td>
<td>Interpreting Categorical &amp; Quantitative Data</td>
<td>8–12</td>
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<td>S-IC</td>
<td>Making Inferences &amp; Justifying Conclusions</td>
<td>Not determined</td>
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<td>S-CP</td>
<td>Conditional Probability &amp; Rules of Probability</td>
<td>Not determined</td>
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<tr>
<td>S-MD</td>
<td>Using Probability to Make Decisions</td>
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### Modeling

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Traditional Pathway

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds upon prior students’ prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from Unit 2, where they extend the laws of exponents to rational exponents. Students strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations and inequalities involving quadratic expressions. Standard N.RN.2 was added to this unit by Maryland educators. This standard deals with simplifying and performing operation on radicals which is a skill that is useful when working with the quadratic formula and later in Geometry. Students learn that some quadratic equations have no real solutions. In Algebra II students will revisit quadratic equations. At that time they will learn to extend the number system to included complex numbers allowing them to determine two solutions for equations such as $x^2 + 1 = 0$.

Critical Area 5: In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. Students explore the effects of translations on the graphs of quadratic functions in preparation for work in Geometry and Algebra II. They will also select the best fitting function

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to model phenomena. Students expand their experience with functions to include more specialized functions—square root, cube root, absolute value, step, and piecewise-defined.

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### Overview of the Units of Study

<table>
<thead>
<tr>
<th>Unit</th>
<th>Clusters</th>
<th>Standards</th>
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</thead>
</table>
| **Unit 1**  
Relationships Between Quantities and Reasoning with Equations | • Reason quantitatively and use units to solve problems.  
• Interpret the structure of expressions.  
• Create equations that describe numbers or relationships.  
• Understand solving equations as a process of reasoning and explain the reasoning.  
• Solve equations and inequalities in one variable. | • N.Q.1, 2,&3  
• A.SSE.1a&1b  
• A.CED.1,2, 3&4  
• A.REI.1&3 |
| **Unit 2**  
Linear and Exponential Relationship | • Solve systems of equations.  
• Represent and solve equations and inequalities graphically.  
• Understand the concept of a function and use function notation.  
• Interpret functions that arise in applications in terms of a context.  
• Analyze functions using different representations.  
• Build a function that models a relationship between two quantities.  
• Build new functions from existing functions.  
• Construct and compare linear and exponential models and solve problems.  
• Interpret expressions for functions in terms of the situation they model. | • A.REI.5, 6,10,11&12  
• F.IF.1,2,3,4, 5,6,7,a,9  
• F.BF.1a,&3  
• F.LE.1a,1b, 1c,2,3&5 |
| **Unit 3**  
Descriptive Statistics | • Summarize, represent, and interpret data on a single count or measurement variable.  
• Summarize, represent, and interpret data on two categorical and quantitative variables.  
• Interpret linear models. | • S.ID.1,2,3, 5,6,7,8&9 |
| **Unit 4**  
Expressions and Equations | • Extend the properties of exponents to rational exponents.  
• Use properties of rational and irrational numbers.  
• Interpret the structure of expressions.  
• Understand the relationship between zeros and factors of polynomials  
• Write expressions in equivalent forms to solve problems.  
• Perform arithmetic operations on polynomials.  
• Create equations that describe numbers or relationships.  
• Solve equations and inequalities in one variable. | • N.RN.2&3  
• A.SSE.1a,1b, 2,3a,3b&3c  
• A.APR.1& 3  
• A.CED.1,2, 5&4  
• A.REI.1, 4a&4b |
| **Unit 5**  
Quadratic Functions and Modeling | • Interpret functions that arise in applications in terms of a context.  
• Analyze functions using different representations.  
• Build a function that models a relationship between two quantities.  
• Build new functions from existing functions.  
• Construct and compare linear, quadratic, and exponential models and solve problems.  
• Summarize, represent, and interpret data on two categorical and quantitative variables | • F.IF.4,5,6,7,a, 7b,8a,&9  
• F.BF.1a&3  
• F.LE.3  
• S.ID.6a |

*Modeling Standard*  
(+ indicates additional mathematics that students should learn to prepare for advanced courses)
Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
Unit 1: Relationships between Quantities and Reasoning with Equations

By the end of eighth grade students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Standard</th>
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<tbody>
<tr>
<td>N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. *</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Working with quantities and the relationships between them provides the groundwork for working with expressions, equations, and functions.

**Essential Skills and Knowledge**
- Ability to choose appropriate units of measure to represent context of the problem
- Ability to convert units of measure using dimensional analysis

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. *

**Essential Skills and Knowledge**
- Ability to select and use units of measure to accurately model a given real world scenario

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *

**Essential Skills and Knowledge**
- Knowledge of and ability to apply rules of significant digits
- Ability to use precision of initial measurements to determine the level of precision with which answers can be reported

* Modeling Standard

(+) indicates additional mathematics that students should learn to prepare for advanced courses
Cluster | Standard
---|---
Interpret the structure of expressions. | Cluster Note: Limit to linear expressions and to exponential expressions with integer exponents.

Note: These are overarching standards that have applications in multiple units.

A.SSE.1  Interpret expressions that represent a quantity in terms of its context. ★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

Essential Skills and Knowledge:
- Ability to make connections between symbolic representations and proper mathematics vocabulary
- Ability to identify parts of an expression such as terms, factors, coefficients, etc.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret \( P(1+r)^n \) as the product of \( P \) and a factor not depending on \( P \).

Essential Skills and Knowledge:
- Ability to interpret and apply rules for order of operations

★ Modeling Standard
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<table>
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</table>
| Create equations that describe numbers or relationships. | A.CED.1 Create equations and inequalities in **one** variable and use them to solve problems. *(Include equations arising from **linear** and **quadratic** functions, and simple rational and **exponential** functions.)*  
**Note:** Limit to linear and exponential relationships, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.  
**Essential Skills and Knowledge**  
- Ability to distinguish between linear and exponential relationships given multiple representations and then create the appropriate equation/inequality using given information  |
| | A.CED.2 Create equations in **two or more** variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  
**Note:** Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.  
**Essential Skills and Knowledge**  
- Ability to distinguish between linear and exponential relationships given multiple representations  
- Ability to determine unknown parameters needed to create an equation that accurately models a given situation  |
| | A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*  
**Note:** Limit to linear equations and inequalities.  
**Essential Skills and Knowledge**  
- Ability to distinguish between a mathematical solution and a contextual solution  |
| | A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law* $V = IR$ *to highlight resistance* $R$.  
**Note:** Limit to formulas which are linear in the variable of interest.  
**Essential Skills and Knowledge**  
- Ability to recognize/create equivalent forms of literal equations  |
### Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a **viable argument** to justify a solution method.

**Note:** Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses.

**Essential Skills and Knowledge**
- Ability to identify the mathematical property (addition property of equality, distributive property, etc.) used at each step in the solution process as a means of justifying a step

### Solve equations and inequalities in one variable.

A.REI.3 Solve linear equations and inequalities in **one** variable, including equations with coefficients represented by letters.

**Note:** Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as \(5^x=125\) or \(2^x=1/16\).

**Essential Skills and Knowledge**
- Ability to analyze the structure of an equation to determine the sequence of steps that need to be applied to arrive at a solution
- Ability to accurately perform the steps needed to solve a linear equation/inequality
Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

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| Solve systems of equations. | A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  
**Note:** Build on student experiences with graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5 when it is taught in Geometry, which requires students to prove the slope criteria for parallel lines. |

**Essential Skills and Knowledge**

- Ability to use various methods for solving systems of equations algebraically
- Ability to identify the mathematical property (addition property of equality, distributive property, etc.) used at each step in the solution process as a means of justifying a step

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Essential Skills and Knowledge**

- Ability to extend experiences with solving simultaneous linear equations from 8EE.8 b&c to include more complex situations
- Ability to solve systems using the most efficient method

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### Represent and solve equations and inequalities graphically.

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<tr>
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<td>A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td>
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<td><strong>Note:</strong> Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.</td>
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<td><strong>Essential Skills and Knowledge</strong></td>
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<td>• Ability to construct an argument as to how the points that make up a curve connect to an algebraic representation of the function that is being represented by the graph</td>
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<td>A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations ( y = f(x) ) and ( y = g(x) ) intersect are the solutions of the equation ( f(x) = g(x) ); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find <strong>successive approximations</strong>. Include cases where ( f(x) ) and/or ( g(x) ) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. <strong>★</strong></td>
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<td><strong>Note:</strong> Focus on cases where ( f(x) ) and ( g(x) ) are linear or exponential.</td>
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<td><strong>Essential Skills and Knowledge</strong></td>
</tr>
<tr>
<td></td>
<td>• Ability to show the equality of two functions using multiple representations</td>
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<td>A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td>
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<td><strong>Essential Skills and Knowledge</strong></td>
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<td>• Ability to explain why a particular shaded region represents the solution of a given linear inequality or system of linear inequalities</td>
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<td></td>
<td>• Ability to convey the mathematics behind the dotted versus solid boundary lines used when graphing the solutions to linear inequalities</td>
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* Modeling Standard

(+) indicates additional mathematics that students should learn to prepare for advanced courses
**Cluster** | **Standard**
--- | ---
Understanding the concept of a function and use function notation. | F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

**Note:** Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.

**Essential Skills and Knowledge**
- Ability to determine if a relation is a function
- Ability to identify the domain and range of a function from multiple representations
- Ability to use of function notation
- Knowledge of and ability to apply the vertical line test

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**Essential Skills and Knowledge**
- Ability to make connections between context and algebraic representations which use function notation

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \) for \( n \geq 1 \).

**Note:** Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.

**Essential Skills and Knowledge**
- See the skills and knowledge that are stated in the Standard.
<table>
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| **Interpret functions that arise in applications in terms of a context.** | F.IF.4 For a function that models a relationship between two quantities, interpret key features of the graph and the table in terms of the quantities, and sketch the graph showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

*Note:* Focus on linear and exponential functions.

**Essential Skills and Knowledge**
- Ability to translate from algebraic representations to graphic or numeric representations and identify key features using the various representations

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

*Note:* Focus on linear and exponential functions.

*Note:* This is an overarching standard that has applications in multiple units.

**Essential Skills and Knowledge**
- Ability to relate the concept of domain to each function studied
- Ability to describe the restrictions on the domain of all functions based on real world context

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

*Note:* Focus on linear functions and exponential functions whose domain is a subset of the integers. Unit 5 in this course and the Algebra II course address other types of functions.

**Essential Skills and Knowledge**
- Knowledge that the rate of change of a function can be positive, negative or zero
- Ability to identify the rate of change from multiple representations

* Modeling Standard

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<table>
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</table>
| Analyze functions using different representations. | Cluster Note: For F.IF.7a, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as \( y = 3^n \) and \( y = 100^n \).  
F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).  
Note: This is an overarching standard that will be visited again in Unit 5 Quadratic Functions and Modeling and in Algebra II. |
| Essential Skills and Knowledge        | • See the skills and knowledge that are stated in the Standard.                                                                                                                                          |
|                                       | Ability to recognize common attributes of a function from various representations.                                                                                                                     |
| Build a function that models a relationship between two quantities. | Cluster Note: Limit F.BF.1a to linear and exponential functions  
F.BF.1 Write a function that describes a relationship between two quantities.  
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.  
Essential Skills and Knowledge  
• See the skills and knowledge that are stated in the Standard. |

* Modeling Standard  
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**F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

**Note:** Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

**Essential Skills and Knowledge**

- See the skills and knowledge that are stated in the Standard.

**F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.

**Essential Skills and Knowledge**

- See the skills and knowledge that are stated in the Standard.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

**Essential Skills and Knowledge**

- Ability to recognize a linear relationship

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

**Essential Skills and Knowledge**

- Ability to recognize an exponential relationship

**F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
### Construct and compare linear, quadratic, and exponential models and solve problems.

**Note:** In constructing linear functions draw on and consolidate previous work on finding equations for lines and linear functions \((8.EE.6, 8.F)\).

**Essential Skills and Knowledge**
- Ability to produce an algebraic model

F.LE.3 Observe using graphs and tables that a quantity increasing **exponentially** eventually exceeds a quantity increasing **linearly**, quadratically, or (more generally) as a polynomial function.

**Note:** Limit to comparisons between linear and exponential models.

**Essential Skills and Knowledge**
See the skills and knowledge that are stated in the Standard.

### Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the **parameters** in a linear or exponential function in terms of a context.

**Note:** Limit exponential functions to those of the form \(f(x) = b^x + k\).

**Essential Skills and Knowledge**
- Ability to interpret the slope and \(y\)-intercept of a linear model in terms of context
- Ability to identify the initial amount present in an exponential model \((f(0)) = b^0 + k = 1 + k\)
- Ability to interpret the rate of increase/decrease in an exponential model

* Modeling Standard

(+): indicates additional mathematics that students should learn to prepare for advanced courses.
Unit 3: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatter plots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

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| Summarize, represent, and interpret data on a single count or measurement variable. | Cluster Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points. 

S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Essential Skills and Knowledge
• Ability to determine the best data representation to use for a given situation
• Knowledge of key features of each plot
• Ability to correctly display given data in an appropriate plot
• Ability to analyze data given in different formats

S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Essential Skills and Knowledge
• Ability to interpret measures of center and spread (variability) as they relate to several data sets
• Ability to identify shapes of distributions (skewed left or right, bell, uniform, symmetric)
• Knowledge that it is appropriate mean/standard deviation for symmetric data versus using the 5 number summary for skewed data

S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Essential Skills and Knowledge
• Ability to recognize gaps, clusters, and trends in the data set
• Ability to recognize extreme data points (outliers) and their impact on center
• Ability to effectively communicate what the data reveals
Knowledge that when comparing distributions there must be common scales and units

* Modeling Standard
(+) indicates additional mathematics that students should learn to prepare for advanced courses
### Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

<table>
<thead>
<tr>
<th>Standard</th>
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<tbody>
<tr>
<td>S.I.D.5</td>
<td>Summarize <strong>categorical data</strong> for two categories in two-way frequency tables. Interpret <strong>relative frequencies</strong> in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</td>
</tr>
</tbody>
</table>
| Essential Skills and Knowledge | - Knowledge of the characteristics of categorical data  
- Ability to read and use a two-way frequency table  
- Ability to use and to compute joint, marginal, and conditional relative frequencies  
- Ability to read a segmented bar graph |
| S.I.D.6 | Represent data on two quantitative variables on a scatter-plot, and describe how the variables are related. |
| a. | Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. |
| Note: | S.I.D.6.a. & c Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals. |
| Essential Skills and Knowledge | - Ability to recognize types of relationships that lend themselves to linear and exponential models  
- Ability to create and use regression models to represent a contextual situation |
| b. | Informally assess the fit of a function by plotting and analyzing residuals. |
| Note: | Focus on linear models, but may use this standard to preview quadratic functions in Unit 5 of this course |
| Essential Skills and Knowledge | - Ability to create a graphic display of residuals  
- Ability to recognize patterns in residual plots  
- Ability to calculate error margins (residuals) with a calculator  
- Ability to analyze the meaning of patterns in residual plots |
| c. | Fit a linear function for a scatter plot that suggests a linear association |
| Essential Skills and Knowledge | - Ability to recognize a linear relationship displayed in a scatter plot  
- Ability to determine an equation for the line of best fit for a set of data points |

* Modeling Standard  
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<tr>
<td>S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</td>
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</table>

**Essential Skills and Knowledge**
- See the skills and knowledge that are stated in the Standard.

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

**Notes:** Build on student experience with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.

**Essential Skills and Knowledge**
- Knowledge of the range of the values \((-1 \leq y \leq 1)\) and the interpretation of those values for correlation coefficients
- Ability to compute and analyze the correlation coefficient for the purpose of communicating the goodness of fit of a linear model for a given data set

S.ID.9 Distinguish between correlation and causation.

**Essential Skills and Knowledge**
- Ability to provide examples of two variables that have a strong correlation but one does not cause the other

* Modeling Standard

(+) indicates additional mathematics that students should learn to prepare for advanced courses
Unit 4: Expressions and Equations

In this unit, students build on their knowledge from Unit 2, where they extend the laws of exponents to rational exponents. Students strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations and inequalities involving quadratic expressions. Standard N.RN.2 was added to this unit by Maryland educators. This standard deals with simplifying and performing operation on radicals which is a skill that is useful when working with the quadratic formula and later in Geometry. Students learn that some quadratic equations have no real solutions. In Algebra II students will revisit quadratic equations. At that time they will learn to extend the number system to included complex numbers allowing them to determine two solutions for equations such as $x^2 + 1 = 0$.

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<tbody>
<tr>
<td>Extend the properties of exponents to rational exponents.</td>
<td>N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.</td>
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<tr>
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<td><strong>Essential Skills and Knowledge</strong></td>
</tr>
<tr>
<td></td>
<td>• Ability to use properties of radicals and rational exponents to transform and simplify radical expressions</td>
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</tbody>
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<tbody>
<tr>
<td>Use properties of rational and irrational Numbers.</td>
<td>N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</td>
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<td><strong>Note:</strong> Connect to physical situations, e.g., finding the perimeter of a square of area 2.</td>
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<td><strong>Essential Skills and Knowledge</strong></td>
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<tr>
<td></td>
<td>• Ability to perform operations on both rational and irrational numbers</td>
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<tr>
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<td>• Make generalizations about sums and products of rational and irrational numbers</td>
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<th>Cluster</th>
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</table>
| **Interpret the structure of expressions.** | Cluster Note: Focus on quadratic and exponential expressions.  
A.SSE.1 Interpret expressions that represent a quantity in terms of its context. ★  
a. Interpret parts of an expression, such as terms, factors, and coefficients.  

**Essential Skills and Knowledge**  
- Ability to extend knowledge of A.SSE.1b from Unit 1 of this course to include quadratic and exponential expressions  

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret \( P(1+r)^n \) as the product of \( P \) and a factor not depending on \( P \).  

**Note:** Exponents are extended from the integer exponents found in Unit 1 to rational exponents focusing on those that represent square or cube roots.  

**Essential Skills and Knowledge**  
- Ability to extend knowledge of A.SSE.1b from Unit 1 of this course to quadratic and exponential expressions  

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see \( 4x^2 - 9y^2 \) as \( (2x)^2 - (3y)^2 \), thus recognizing it as a difference of squares that can be factored as \( (2x - 3y)(2x + 3y) \).  

**Note:** This is an overarching standard that has applications in multiple units.  

**Essential Skills and Knowledge**  
- Ability to use properties of mathematics to alter the structure of an expression  
- Ability to select and then use an appropriate factoring technique  

★ Modeling Standard  
(+)) indicates additional mathematics that students should learn to prepare for advanced courses
<table>
<thead>
<tr>
<th>Cluster</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write expressions in equivalent</td>
<td>Cluster Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.</td>
</tr>
<tr>
<td>forms to solve problems.</td>
<td>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★</td>
</tr>
<tr>
<td></td>
<td>a. Factor a quadratic expression to reveal the zeros of the function it defines.</td>
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<tr>
<td></td>
<td><strong>Essential Skills and Knowledge</strong></td>
</tr>
<tr>
<td></td>
<td>• Ability to connect the factors, zeros and x-intercepts of a graph</td>
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<tr>
<td></td>
<td>• Ability to use the Zero-Product Property to solve quadratic equations</td>
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<tr>
<td></td>
<td>• Ability to recognize that quadratics that are perfect squares produce graphs which are tangent to the x-axis at the vertex</td>
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<tr>
<td></td>
<td>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</td>
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<tr>
<td></td>
<td><strong>Essential Skills and Knowledge</strong></td>
</tr>
<tr>
<td></td>
<td>• Ability to recognize key features of a quadratic model given in vertex form</td>
</tr>
<tr>
<td></td>
<td>c. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} ≈ 1.01212t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</td>
</tr>
<tr>
<td></td>
<td><strong>Essential Skills and Knowledge</strong></td>
</tr>
<tr>
<td></td>
<td>• Ability to connect experience with properties of exponents from Unit 2 of this course to more complex expressions</td>
</tr>
</tbody>
</table>
### Cluster: Perform arithmetic operations on polynomials

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are **closed** under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**Note:** Limit to linear and quadratic polynomials

**Essential Skills and Knowledge**
- Ability to show that when polynomials are added, subtracted or multiplied that the result is another polynomial

### Cluster: Understand the relationship between zeros and factors of polynomials

A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**Essential Skills and Knowledge**
- Ability to identify the zeros of a cubic polynomial of the form (linear factor)(factorable quadratic factor)
### Cluster: Create equations that describe numbers or relationships.

#### Standard: A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and **quadratic** functions, and simple rational and exponential functions.

**Essential Skills and Knowledge**
- Ability to distinguish between linear, quadratic and exponential relationships given the verbal, numeric and/or graphic representations.

#### Standard: A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**Essential Skills and Knowledge**
- Ability to distinguish between linear, quadratic and exponential relationships given numeric, or verbal representations.
- Ability to determine unknown **parameters** needed to create an equation that accurately models a given situation.

#### Standard: A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law* $V = IR$ to highlight resistance $R$.

**Note:** Extend to formulas involving squared variables.

**Essential Skills and Knowledge**
- Ability to recognize and create different forms of literal equations.

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### Cluster: Understanding solving equations as a process of reasoning and explain with reasoning.

#### Standard: A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a **viable argument** to justify a solution method.

**Essential Skills and Knowledge**
- Ability to identify the mathematical property (addition property of equality, distributive property, etc.) used at each step in the solution process as a means of justifying a step.

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* Modeling Standard

(+) indicates additional mathematics that students should learn to prepare for advanced courses.
Cluster Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

A.REI.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Essential Skills and Knowledge
- Ability to solve literal equations for a variable of interest

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula reveals that the quadratic equation has "no real solutions".

Essential Skills and Knowledge
- Ability to solve quadratic equations using various methods and recognize the most efficient method
- Ability to use the value of the discriminant to determine if a quadratic equation has one double solution, two unique solutions or no real solutions
Unit 5: Quadratic Functions and Modeling

In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. Students explore the effects of translations on the graphs of quadratic functions in preparation for work in Geometry and Algebra II. They will also select the best fitting function to model phenomena. Students expand their experience with functions to include more specialized functions—square root, cube root, absolute value, step, and piecewise-defined.

<table>
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| **Interpret functions that arise in applications in terms of a context.** | Cluster Note: Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2  
Note: These are overarching standards that will be revisited for each function studied but as each new function is introduced, modeling problems should not be limited to just the newly introduced function but should include all functions studied.  
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★  
**Essential Skills and Knowledge**  
- Ability to connect experiences with linear and exponential functions from Unit 2 of this course to quadratic, square root, cube root, absolute value, step and piecewise defined models.  
- Ability to connect appropriate function to context  
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.★  
**Essential Skills and Knowledge**  
- Ability to connect experiences with linear and exponential functions from Unit 2 of this course to quadratic, square root, cube root, absolute value, step and piecewise defined models.  
- Ability to describe the restrictions on the domain of a function based on real world context  
- Ability to recognize and use alternate vocabulary for domain and range such as input/output or independent/dependent |

★ Modeling Standard

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### Interpret functions that arise in applications in terms of a context.

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<tbody>
<tr>
<td>F.IF.6</td>
<td>Calculate and interpret the <strong>average rate of change</strong> of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
</tr>
</tbody>
</table>

**Essential Skills and Knowledge**
- Knowledge that the rate of change of a function can be positive, negative, zero or can have no change
- Ability to identify the rate of change from multiple representations

### Analyze functions using different representations.

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<tr>
<td>Cluster Note: This unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.</td>
<td></td>
</tr>
<tr>
<td>F.IF.7</td>
<td>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</td>
</tr>
</tbody>
</table>
- a. Graph linear and **quadratic** functions and show intercepts, **maxima**, and **minima**. |

**Essential Skills and Knowledge**
- Ability to connect experience with graphing linear functions from Unit 2 of this course to include quadratic functions

- b. Graph square root, cube root, and **piecewise-defined** functions, including **step** functions and **absolute value** functions. |

**Note:** Compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions

**Essential Skills and Knowledge**
- Ability to make a quick sketch of each **parent function** over the set of real numbers
- Ability to make connections between a function’s domain and range and the appearance of the graph of the function
- Knowledge of how parameters introduced into a function alter the shape of the graph of the **parent function**
### Analyze functions using different representations

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<tr>
<td>F.IF.8</td>
<td>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</td>
</tr>
<tr>
<td>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</td>
<td></td>
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</tbody>
</table>

**Essential Skills and Knowledge**
- Ability to make connections between different algebraic representations, a graph and a contextual model
- Ability to recognize common attributes of a function from multiple representations

| F.IF.9  | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions) |
| Note: Focus on expanding the types of functions considered to include, linear, exponential, and quadratic. |

**Essential Skills and Knowledge**
- Ability to connect experience with comparing linear and exponential functions from Unit 2 of this course to include quadratic functions
- Ability to recognize common attributes of a function from multiple representations

### Build a function that models a relationship between two quantities

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<tr>
<td>Cluster Note: Focus on situations that exhibit a quadratic relationship.</td>
<td></td>
</tr>
<tr>
<td>F.BF.1</td>
<td>Write a function that describes a relationship between two quantities.</td>
</tr>
<tr>
<td>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</td>
<td></td>
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</table>

**Essential Skills and Knowledge**
- Ability to connect experience with linear and exponential functions from Unit 2 of this course to quadratic functions
- Ability to write the algebraic representation of a quadratic function from a contextual situation

* Modeling Standard

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| **Build new functions from existing functions.** | F.BF.3 Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.  
**Note:** Focus on quadratic functions, and consider including absolute value functions.  
**Note:** This is an overarching standard that has applications in multiple units in this course.  
**Essential Skills and Knowledge**  
- Ability to make generalizations about the changes that will result in the graph of any function as a result of making a particular change to the algebraic representation of the function |

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| **Construct and compare linear, quadratic, and exponential models and solve problems.** | F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.  
**Note:** Compare linear and exponential growth to quadratic growth  
**Note:** This is an overarching standard that has applications in multiple units in this course.  
**Essential Skills and Knowledge**  
- Ability to recognize linear, quadratic and exponential relationships |
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| Summarize, represent, and interpret data on two categorical and quantitative variables. | S.ID.6 represent data on two quantitative variables on a scatter-plot, and describe how the variables are related.  
  
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, exponential and quadratic models.*  

**Essential Skills and Knowledge**  
- Ability to fit a quadratic function to a set of data in a modeling context  
- Ability to identify which model is most appropriate; linear, exponential or quadratic