Algebra II

Maryland College and Career Ready Standards for Mathematics

Number and Quantity (N)

N.RN The Real Number System

N.RN.A EXTEND THE PROPERTIES OF EXPONENTS TO RATIONAL EXPONENTS.

N.RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $\left(5^{\frac{1}{3}}\right)^3 = 5^{\left(\frac{1}{3}\right)3}$ to hold, so $5^{\left(\frac{1}{3}\right)3}$ must equal 5.

N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N.Q Quantities

N.Q.A REASON QUANTITATIVELY AND USE UNITS TO SOLVE PROBLEMS.

N.Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N.CN The Complex Number System

N.CN.A PERFORM ARITHMETIC OPERATIONS WITH COMPLEX NUMBERS.

N.CN.A.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.

N.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.C USE COMPLEX NUMBERS IN POLYNOMIAL IDENTITIES AND EQUATIONS.

N.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

Algebra (A)

A.SSE Seeing Structure in Expressions

A.SSE.A INTERPRET THE STRUCTURE OF EXPRESSIONS.

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 - y^2)$.

A.SSE.B WRITE EXPRESSIONS IN EQUIVALENT FORMS TO SOLVE PROBLEMS.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A.SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

A.APR Arithmetic with Polynomials and Rational Expressions

A.APR.B UNDERSTAND THE RELATIONSHIP BETWEEN ZEROS AND FACTORS OF POLYNOMIALS.

A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $\boldsymbol{p(x)}$ and a number a, the remainder on division by $x - a$ is $p(a)$ so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available; use the zeros to construct a rough graph of the function defined by the polynomial.

A.APR.C USE POLYNOMIAL IDENTITIES TO SOLVE PROBLEMS.

A.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A.APR.D REWRITE RATIONAL EXPRESSIONS.

A.APR.D.6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.CED Creating Equations

A.CED.A CREATE EQUATIONS THAT DESCRIBE NUMBERS OR RELATIONSHIPS.

A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.REI Reasoning with Equations and Inequalities

A.REI.A UNDERSTAND SOLVING EQUATIONS AS A PROCESS OF REASONING AND EXPLAIN THE REASONING.

A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.B SOLVE EQUATIONS AND INEQUALITIES IN ONE VARIABLE.

A.REI.B.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

A.REI.C SOLVE SYSTEMS OF EQUATIONS.

A.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

A.REI.D REPRESENT AND SOLVE EQUATIONS AND INEQUALITIES GRAPHICALLY.

A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Functions (F)

F.IF Interpreting Functions

F.IF.A UNDERSTAND THE CONCEPT OF A FUNCTION AND USE FUNCTION NOTATION.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.

F.IF.B INTERPRET FUNCTIONS THAT ARISE IN APPLICATIONS IN TERMS OF THE CONTEXT.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C ANALYZE FUNCTIONS USING DIFFERENT REPRESENTATIONS.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = \frac{(1.2)^t}{10}$, and classify them as representing exponential growth or decay.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.

F.BF Building Functions

F.BF.A BUILD A FUNCTION THAT MODELS A RELATIONSHIP BETWEEN TWO QUANTITIES.

F.BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.BF.B BUILD NEW FUNCTIONS FROM EXISTING FUNCTIONS.

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.B.4 Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = \frac{x+1}{x-1}$ *for* $x \neq 1$.

F.LE Linear, Quadratic, and Exponential Models

F.LE.A CONSTRUCT AND COMPARE LINEAR, QUADRATIC, AND EXPONENTIAL MODELS AND SOLVE PROBLEMS.

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.4 For exponential models, express as a logarithm the solution to $\boldsymbol{ab^{ct} = d}$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

F.LE.B INTERPRET EXPRESSIONS FOR FUNCTIONS IN TERMS OF THE SITUATION THEY MODEL.

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

F.TF Trigonometric Functions

F.TF.A EXTEND THE DOMAIN OF TRIGONOMETRIC FUNCTIONS USING THE UNIT CIRCLE.

F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

S.TF.B MODEL PERIODIC PHENOMENA WITH TRIGONOMETRIC FUNCTIONS.

F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.C PROVE AND APPLY TRIGONOMETRIC IDENTITIES.

F.TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Statistics (S)

S.ID Interpreting categorical and quantitative data

S.ID.B SUMMARIZE, REPRESENT, AND INTERPRET DATA ON TWO CATEGORICAL AND QUANTITATIVE VARIABLES.

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.