Grade 4 Mathematics

Maryland College and Career Ready Curriculum Framework

Introduction

The Code of Maryland Regulations (COMAR) 13A.04.12.01, Mathematics Instructional Programs for Grades Prekindergarten – 12 states that, "each local education agency shall provide in public schools an instructional program in mathematics each year for all students in grades prekindergarten – 8; Offer in public schools a mathematics program in grades 9—12. Beginning with students entering grade 9 in the 2014—2015 school year, each student shall enroll in a mathematics course in each year of high school that the student attends, up to a maximum of 4 years of attendance, unless in the 5th or 6th year a mathematics course is needed to meet a graduation requirement."

State Frameworks are developed by the Maryland State Department of Education (MSDE) to support local education agencies in providing high-quality instructional programs in mathematics. State Frameworks are defined as supporting documents and provide guidance for implementing the Maryland College and Career Ready Standards for Mathematics which are reviewed and adopted by the Maryland State Board of Education every eight years. State Frameworks also provide consistency in learning expectations for students in mathematics programs across the twenty-four local education agencies as local curriculum is developed and adopted using these documents as a foundation.

MSDE shall update the State Frameworks in Mathematics in the manner and time the State Superintendent of Schools determines is necessary to ensure alignment with best-in-class, research-based practices. Tenure and stability of State Frameworks affords local education agencies the necessary time to procure supporting instructional materials, provide professional development, and to measure student growth within the program. Educators, practitioners, and experts who participate in writing workgroups for State Frameworks represent the diversity of stakeholders across Maryland. State Frameworks in Elementary mathematics grades Prekindergarten – 5 were developed, reviewed, and revised by teams of Maryland educators and practitioners, including local education agency content curriculum specialists, classroom teachers, accessibility staff, and academic researchers and experts in close collaboration with MSDE.

The Grade 4 Mathematics Framework was released in June 2011.

HOW TO READ THE MARYLAND COLLEGE AND CAREER READY CURRICULUM FRAMEWORK

The Maryland College and Career Ready Standards for Mathematics (MCCRSM) at the fourth-grade level specify the mathematics that all students should study as they prepare to be college and career ready by graduation. The fourth-grade standards are listed by domains. For further clarification of the standards, reference the appropriate domain in the set of Progression Documents for the Common Core State Standards for Mathematics or the Grade 4 Mathematics Evidence Statements located on the MSDE MCAP Mathematics webpage.

This framework document provides an overview of the Standards that are grouped together to form the domains for grade one. The Standards within each domain are grouped by topic and are in the same order as they appear in the Common Core State Standards for Mathematics. This document is not intended to convey the exact order in which the Standards will be taught, nor the length of time to devote to the study of the different standards.

The framework contains the following:

- **Domains** are intended to convey coherent groupings of content.
- **Clusters** are groups of related standards.
- **Standards** define what students should understand and be able to do.
- **Essential Skills and Knowledge** statements provide language to help teachers develop common understandings and valuable insights into what a student must know and be able to do to demonstrate proficiency with each standard. Maryland mathematics educators thoroughly reviewed the standards and, as needed, provided statements to help teachers comprehend the full intent of each standard.
- **Framework Vocabulary Words** provide definitions of key mathematics vocabulary words found in the document.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. REASON ABSTRACTLY AND QUANTITATIVELY.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify to improve the arguments.

4. MODEL WITH MATHEMATICS.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. USE APPROPRIATE TOOLS STRATEGICALLY.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. ATTEND TO PRECISION.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. LOOK FOR AND MAKE USE OF STRUCTURE.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

8. LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1)$ equals 3. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE MARYLAND COLLEGE AND CAREER READY STANDARDS

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

4.OA Operations and Algebraic Thinking

4.OA.A USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS.

4.OA.A.1

Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Essential Skills and Knowledge

- Knowledge of and ability to apply understanding of multiplication as repeated addition (2.OA.C.4), as "equal groups of" (3.OA.A.1), and the Commutative Property (3.OA.B.5).
- Ability to identify a multiplication equation as a comparison.
- Ability to verbally explain which quantity is being multiplied and which number tells how many times as multiplication equations.

4.OA.A.2

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Essential Skills and Knowledge

- Ability to solve various types of word problems involving multiplication comparison by using drawings (Table 2) through initial use of concrete materials and pictures, leading to the use of equations as a tool in solutions.
- Ability to represent the solution to multiplicative comparison problems using multiplication or division equations.
- Ability to distinguish and explain the difference between multiplicative comparison and additive comparison.

4.OA.A.3

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Essential Skills and Knowledge

- Ability to identify which of the four operations will be used to solve multi-step word problems and accurately represent the problem with the corresponding equations.
- Ability to use the context of the problem to interpret the remainder of a problem to appropriately determine if it should be discarded, replaced with the next highest whole number answer, or used as the answer to the question.

- Ability to use mental math strategies, properties of operations, relationships between operations, and estimation strategies to solve multistep word problems.
- Ability to verbally and in writing provide evidence of reasonableness of answers using mental computation, properties of operations, relationships between operations, and estimation strategies.

4.OA.B GAIN FAMILIARITY WITH FACTORS AND MULTIPLES.

4.OA.B.4

Find all factor pairs for a whole number in the range 1–100.Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range of 1–100 is prime or composite.

Essential Skills and Knowledge

- Knowledge of multiplication as arrays and its connection to area of rectangles to determine factor pairs.
- Knowledge of and ability to apply multiplication facts to determine multiples of one-digit numbers.

4.OA.C GENERATE AND ANALYZE PATTERNS.

4.OA.C.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Essential Skills and Knowledge

- Ability to apply knowledge of growing patterns versus repeating patterns using either numbers or shapes.
- Ability to analyze a set of numbers to identify the pattern given the rule.
- Ability to extend the pattern based on the rule given or identified.
- Ability to provide an explanation why numbers will continue an identified pattern.

4.NBT Number and Operations in Base Ten

4.NBT.AGENERALIZE PLACE VALUE UNDERSTANDING FOR MULTI-DIGIT WHOLE NUMBERS.

4.NBT.A.1
Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

Essential Skills and Knowledge

- Ability to extend knowledge of place value from prior grades (2.NBT.A.1-4, 3.NBT.A.3).
- Ability to extend their knowledge of place value related to multiplying and dividing by multiples of 10.
- Ability to reason and explain the magnitude of the digits in a number, for example How is the 8 in the number 685 similar or different to the number 8 in 658.

4.NBT.A.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Essential Skills and Knowledge

- Ability to flexibly read and write different number forms, e.g., base ten numerals (285), extended form (200 + 80 + 5), and written form (two hundred eight-five).
- Compare two multi-digit numbers by value of the digits in each place using comparison symbols.

4.NBT.A.3
Use place value understanding to round multi-digit whole numbers to any place.

Essential Skills and Knowledge

- This standard requires students to demonstrate the ability to round multi-digit numbers to any place, but goes beyond the procedure of rounding to explain how they rounded.
- Ability to use their understanding of place value to reason and explain the answer for a rounding problem.

4.NBT.BUSE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC.

4.NBT.B.4
Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Essential Skills and Knowledge

- Knowledge of various types of algorithms to perform multi-digit arithmetic.
- Ability to apply a standard algorithm in both addition and subtraction problems.

4.NBT.B.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Essential Skills and Knowledge

- Apply to apply knowledge of multiplication and division facts of one-digit whole numbers.
- Knowledge of and ability to apply the Properties of Operations (Table 3).
- Knowledge of and ability to apply understanding of place value when multiplying multi-digit numbers.

4.NBT.B.6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Essential Skills and Knowledge

- Ability to apply knowledge of multiplication and division within 100 (3. OA.C.7).
- Ability to use arrays and area models for multiplication and division (3. MD.C.6 and 3.MD.C.7).
- Knowledge of and ability to apply the Properties of Operations (Table 3).

4.NF Number and Operations – Fractions

4.NF.A EXTEND UNDERSTANDING OF FRACTION EQUIVALENCE AND ORDERING.

4.NF.A.1

Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Essential Skills and Knowledge

- Ability to use concrete materials to model fraction number concepts and values.
- Knowledge of and ability to generate simple equivalent fractions (3. NF.A.3b).
- Extend work from third grade by using additional denominators 5,10,12, and 100.
- Generate a rule for finding equivalent fractions based on conceptual understanding of using models to show equivalent fractions.

4.NF.A.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, <, and justify the conclusion, e.g., by using a visual fraction model.

Essential Skills and Knowledge

- Ability to apply knowledge factors (4. OA.B.4) to the strategies used to determine equivalent fractions as well as ordering fractions.
- Ability to apply reasoning such as $\frac{5}{12} < \frac{1}{2}$ because $\frac{6}{12}$ is equivalent to one half so five twelfths is less than one half.
- Ability to identify the 'whole' for the fractions being compared.
- Ability to compare unlike fractions as stated in this Standard lays the foundation for knowledge of strategies such as finding the Least Common Multiple or the Greatest Common Factor.

4.NF.B BUILD FRACTIONS FROM UNIT FRACTIONS BY APPLYING AND EXTENDING PREVIOUS UNDERSTANDINGS OF OPERATIONS ON WHOLE NUMBERS.

4.NF.B.3

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

4.NF.B.3a

Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Essential Skills and Knowledge

- Ability to use concrete and/or pictorial tools to add and subtract fractions with like denominators.
- Ability to use manipulatives or drawings to demonstrate the denominator does not change when adding or subtracting fractions with like denominators.
- Knowledge that the numerator tells how many parts of the whole we are counting and the denominator tells how many total parts there are in all.
- Ability to represent the addition and subtraction of fractions using concrete materials, pictures, numbers, and words.

4.NF.B.3b

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition as an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ or $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$ or $2\frac{1}{8} = 1 + 1 + \frac{1}{8}$ or $2\frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

Essential Skills and Knowledge

- Ability to represent a whole number as a fraction.
- Ability to decompose fractions greater than one into whole numbers and fractional parts (3.NF.A.3c).

4.NF.B.3c

Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Essential Skills and Knowledge

- Ability to change a mixed number into an improper fraction.
- Ability to add mixed numbers using a strategy such as adding.
- fractions together and then adding the whole numbers together.
- Ability to subtract mixed numbers using a strategy such as replacing each mixed number with an equivalent fraction and then subtracting.

4.NF.B.3d

Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Essential Skills and Knowledge

- Ability to apply the understanding that the numerator tells us how many parts of the whole we are counting and the denominator tells us how many total parts there are in the whole.

4.NF.B.4

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Essential Skills and Knowledge

- Ability to use concrete materials to model multiplication of fractions.
- Knowledge that when multiplying a whole number by a fraction, you are finding that fractional part of the whole number (e.g., $\frac{1}{4} \times 24$ is the same as $\frac{1}{4}$ of 24).
- Ability to connect the multiplication of fractions to the repeated addition of fractions (e.g., $4 \times \frac{2}{4} = \frac{2}{4} + \frac{2}{4} + \frac{2}{4} + \frac{2}{4}$).

4.NF.B.4a

Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product of* $5 \times \frac{1}{4}$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \times \frac{1}{4}$.

Essential Skills and Knowledge

- Ability to apply the concept of a unit fraction in relationship to a multiple of that fraction (e.g., $\frac{1}{4}$ is the unit fraction of fourths).

4.NF.B.4b

Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times \frac{2}{5}$ *as* $6 \times \frac{1}{5}$, *recognizing this product as* $\frac{6}{5}$. *(In general,* $n \times \frac{a}{b} = \frac{n \times a}{b}$.*)*

Essential Skills and Knowledge

- Knowledge that $3 \times \frac{2}{5} = 3$ groups of $\frac{2}{5}$ or $\frac{2}{5} + \frac{2}{5} + \frac{2}{5}$.

4.NF.B.4c

Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if a person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Essential Skills and Knowledge

- Ability to apply knowledge of multiplication of fractions by a whole number to a variety of real life problem situations.

4.NF.C UNDERSTAND DECIMAL NOTATION FOR FRACTIONS, AND COMPARE DECIMAL FRACTIONS.

4.NF.C.5

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, when adding* $\frac{3}{10}$ *and* $\frac{4}{100}$ *express* $\frac{3}{10}$ *as* $\frac{30}{100}$, *then* $\frac{30}{100}+\frac{4}{100}=\frac{34}{100}$.

Essential Skills and Knowledge

- Knowledge of this standard provides a foundation for the relationship between fractions and decimals.
- Knowledge of relationship dimes and dollars.
- Ability to use place value blocks and grid paper to show and explain the equivalence.

4.NF.C.6

Use decimal notation for fractions with denominators 10 and 100. *For example, rewrite 0.62 as* $\frac{62}{100}$ *or describe a length as 0.62 meters or locate 0.62 on a number line diagram.*

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the Standard and in standard 4. NF.C.5.

4.NF.C.7

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, <, and justify the conclusions, e.g., by using a visual model.

Essential Skills and Knowledge

- Ability to apply knowledge of place value as a strategy to compare decimals.

4.MD Measurement and Data

4.MD.A SOLVE PROBLEMS INVOLVING MEASUREMENT AND CONVERSION OF MEASUREMENTS FOR A LARGER UNIT TO A SMALLER UNIT.

4.MD.A.1

Know relative sizes of measurement units within one system of units including km, m, cm, kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), etc.*

Essential Skills and Knowledge

- Knowledge of capacity units should also include cups, pints, quarts, and gallons.
- Knowledge of length units should also include inches, feet, and yards.
- Ability to use visual aids with conversion of measurement.

4.MD.A.2

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Essential Skills and Knowledge

- Ability to use visual aids with conversion of measurement.
- Knowledge of systems of measurement, fractions, decimals, and equivalent units of measurement.

4.MD.A.3

Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formulas as a multiplication equation with an unknown factor.*

Essential Skills and Knowledge

- Ability to apply knowledge of the relationship between area and perimeter through the exploration of rectangles with the same area but different perimeters or rectangles with the same perimeter but different areas.
- Ability to apply knowledge of factors, finding an unknown factor in an equation, and the relationship between multiplication and area.

4.MD.B REPRESENT AND INTERPRET DATA.

4.MD.B.4

Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the Standard.
- See the Vocabulary section in this Framework document for an example of a line plot.

4.MD.C GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OR ANGLE AND MEASURE ANGLES.

4.MD.C.5

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

4.MD.C.5a

An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of the circle is called a "one-degree angle," and can be used to measure angles.

Essential Skills and Knowledge

- Knowledge of partitioning circles into equal shares (2.G.A.3).
- Ability to relate understanding of equal shares of a circle to angles.
- Ability to use visual aids and/or technology to apply the understanding of how a circle is divided into 360 degrees (e.g., circle protractor or geometry software).
- Introduce the unit of measurement of a circle (degrees). Students need to understand that a whole circle is 360 degrees by taking a circle and dividing it into $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ etc. so that $\frac{1}{2}$ is 360 divided by 4, etc.

4.MD.C.5b

An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

Essential Skills and Knowledge

- Knowledge that each angle measure is a result of how much of the circle is covered (e.g., shading in 50 parts of the 360 would equal a 50-degree angle)

4.MD.C.6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the standard.

4.MD.C.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Essential Skills and Knowledge

- Ability to apply knowledge of common whole number addition and subtraction situations to fractional problem situations (Table 1).
- Ability to use manipulatives to model the solution to the problem.

4.G Geometry

4.G.A DRAW AND IDENTIFY LINES AND ANGLES, AND CLASSIFY SHAPE BY PROPERTIES OF THEIR LINES AND ANGLES.

4.G.A.1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Essential Skills and Knowledge

- This is the first time these terms are introduced.
- Ability to apply a deep understanding of this vocabulary will assist with drawing and identifying these shapes within two- dimensional figures.

4.G.A.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Essential Skills and Knowledge

- Ability to use concrete materials to model the lines and angles of two-dimensional figures to provide visual evidence of the relationship between various figures.

4.G.A.3
Recognize a line of symmetry for a two- dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the Standard.

Grade 4 MD College and Career-Ready Vocabulary

MULTIPLICATIVE COMPARISON

Comparing the size of a product to the size of one factor on the basis of the size of the other factor. Example: If 2 bags of candy cost $4.00 then 8 bags of candy would cost $16.00

EQUATION

a number sentence stating that the expressions on either side of the equal sign are, in fact, equal?

ADDITIVE COMPARISON

Comparing the size of a sum to the size of one of addend on the basis of the size of the other addend. Example: Johnny is 10 years old. His brother, Sean is 6 years old. When Johnny is 16 years old, Sean will 12 years old.

FACTOR PAIRS

Two numbers that when multiplied equal a product. Examples of factor pairs for the number are 2 and 6; 3 and 4; 1 and 12.

MULTIPLE

the product of a whole number and any other whole number. Example:20 is a multiple of 5 because $4 \times 5 = 20$.

PRIME NUMBER

a whole number greater than 1 that has exactly two factors, 1 and the number itself.

COMPOSITE NUMBER

a whole number that has more than two factors.

APPARENT FEATURES OF PATTERN

identifying features of a pattern that are not explicit in the rule itself. For example, when skip counting beginning with 5, the ones digit is always a 5 or 0. However, when skip counting by 5 beginning with 3, the ones digit is always a 3 or 8.

PATTERN

a repetition of an attribute in a predicable sequence (applies to things found in nature, shapes, numbers, etc.)

GROWING PATTERN

a pattern that increases or decreases by a constant difference from the original core

REPEATING PATTERN

a pattern in which a pattern core is repeated and never changes, nothing is added to or removed from the initial pattern core. For example, a pattern of shapes could be a triangle, hexagon, circle, triangle hexagon circle, triangle, hexagon, circle. Etc.

PLACE VALUE

the value of a digit as determined by its position in a number. For example: in the number "101" the one is worth either 100 or 1, depending upon its position.

BASE TEN NUMERALS

a base of a numeration system is the number that is raised to various powers to generate the place values of that system. In the base ten numeration system the base is ten. The first place is 10^0 or 1 (the units place), the second is 10^1 or 10 (the tens place), the third is 10^2 or 100 (the hundreds place), etc. This determines the place value of the different positions in a number.

NUMBER NAMES

base ten numerals written in word form. Example: 6,427 would be written as six thousand four hundred twenty-seven.

EXPANDED FORM

a numeral expressed as a sum of the products of each digit and its place value. Example: 6,427 written in expanded form would be 6,000 + 400 + 20 + 7.

MULTI-DIGIT WHOLE NUMBERS

a whole number comprised more than one digit. Example: 27 and 246,910 are both multi-digit whole numbers.

STANDARD ALGORITHM

an algorithm is a systematic scheme for performing computations, consisting of a set of rules or steps.

RECTANGULAR ARRAYS

the arrangement of counters, blocks, or graph paper squares in a rectangle made up of rows and columns to represent a multiplication or division equation. For example, two rows of four stars equal eight stars.

AREA MODEL

a drawing or diagram to represent an area problem, where the length and width represent the factors. The model may be a rectangular array of square units, arranged in rows and columns, or a diagram with given dimensions

EQUIVALENT FRACTIONS

two or more fractions that have the same value and the same size whole.

VISUAL FRACTION MODEL

a model that shows operations or properties of fractions using concrete objects or pictures. Examples are fraction strips, bar diagrams, number lines, Cuisenaire rods

DECIMAL

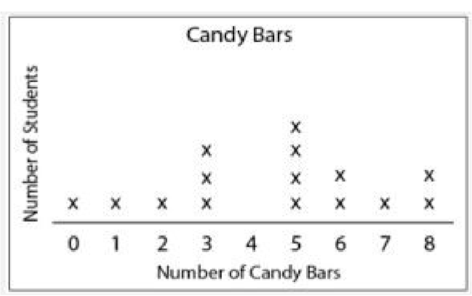
a fraction with unwritten denominator of 10 or some power of 10, indicated by a point (.) before the numerator.

DECOMPOSE

breaking a number into two or more parts to make it easier with which to work. For example, the number eight can be decomposed into three and five or four and four, or seven and one. Decomposing numbers into its parts can help students mentally add numbers. When addition eight and seven students may mentally think eight is composed of seven and one. I know seven and seven equals fourteen, add the one and that equals fifteen.

LINE PLOT

Is a graph that shows frequency of data along a number line. It is best to use a line plot when comparing fewer than 25 numbers. It is a quick, simple way to organize data.



POINT

a position in space

LINE

a set of points that extend infinitely in two opposite directions.

LINE SEGMENT

a subset of a line bounded by two endpoints.

RAY

a subset of a line that has one endpoint and extends infinitely in one direction.

ANGLE

a figure formed by two rays that have the same endpoint. Types of angles include acute, right, obtuse, and reflex angles. Angles are measured in degrees.

RIGHT ANGLE

an angle that has a measurement of 90°.

ACUTE ANGLE

an angle that has a measurement of less than 90°.

OBTUSE ANGLE

an angle that has a measurement greater than 90° but less than 180°

PERPENDICULAR LINES

two lines that intersect to form right angles.

PARALLEL LINES

two lines on a plane that do not intersect.

TWO-DIMENSIONAL FIGURES

A geometric figure that lies entirely in one plan (also called a plane figure).

LINE OF SYMMETRY

A line that divides a figure into two halves that are mirror images of each other. Each point in one of the halves of the figure is the same distance from the line of symmetry as the corresponding point in the other half. A figure may have any number of lines of symmetry. For example, a parallelogram that is not a rectangle has no lines of symmetry. A square has four lines of symmetry horizontal, vertical and diagonal. A circle has infinitely many lines of symmetry.

Table 1: Common addition and subtraction situations.

	Results Unknown	Change Unknown	Start Unknown
Add to	Two birds sat on a ledge. Three more birds flew to the ledge. How many birds are now on the ledge? $2+3=?$	Two birds sat on a ledge. Some more birds flew to the ledge. Then there were five birds on the ledge. How many birds flew over to the first two? $2+?=5$	Some birds sat on a ledge. Three more birds flew to the ledge. Then there were five birds on the ledge. How many birds were on the ledge before? $?+3=5$
Take From	Three oranges were on the table. I ate one orange. How many oranges are on the table now? $3-1=?$	Three oranges were on the table. I ate some oranges. Then there were two oranges. How many oranges did I eat? $3-?=2$	Some oranges were on the table. I ate one orange. Then there were two oranges. How many oranges were on the table before? $?-3=2$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together / Take Apart	Five red marbles and two green marbles are on the table. How many marbles are on the table? $5+2=?$	Ten marbles are on the table. Five are red and the rest are green. How many marbles are green? $5+?=10$ or $?+5=10$	Max has five marbles. How many can she put in her left hand and how many in her right hand? $5=0+5$ $5=5+0$ $5=1+4$ $5=4+1$ $5=2+3$ $5=3+2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare "more"	**"How many more?" version:** Macy has two cats. Marcus has five cats. How many more cats does Marcus have than Macy? $2+?=5$	**Version with "more":** Marcus has three more cats than Macy. Macy has two cats. How many cats does Marcus have? $2+3=?$	**Version with "more":** Marcus has three more cats than Macy. Marcus has five cats. How many cats does Macy have? $5-3=?$
Compare "fewer"	**"How many fewer?" version:** Macy has two cats. Marcus has five cats. How many fewer cats does Macy have than Marcus? $5-2=?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Macy has two cats. How many cats does Marcus have? $3+2=?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Marcus has five cats. How many cats does Macy have? $?+3=5$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes that students should work on in grade 1 but need not master until grade 2.

Adapted from CCSS, p.88, which is based on Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity, National Research Council, 2009, pp. 32-22 and the CCSS Progression document pp. 9.

Table 2: Common multiplication and division situations.

Problem Situation	Unknown Product 3 × 6 = ?	Group Size Unknown (How many in each group?) 3 × ? = 18 and 18 ÷ 3 = ?	Number of Groups Unknown (How many groups?)
Equal groups (Grades 3 – 5)	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: Alex needs 3 lengths of string, each 6 inches long. How much string will Alex need?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: Dale has 18 inches of string, which he will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: Ruth has 18 inches of string, which she will cut into pieces that are 6 inches long. How many pieces of string will she have?
Arrays and Area (Grades 3 – 5)	There are 3 rows of apples with 6 in each row. How many apples are there? OR The apples in the grocery window are in 3 rows and 6 columns. How many apples are there? Area Example: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? OR If 18 apples are arranged into an array with 3 rows, how many columns of apples are there? Area example: A rectangle has an area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? OR If 18 apples are arranged into an array with 3 columns, how many rows are there? Area example: A rectangle has an area of 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare (Grades 4 – 5) Multiplicative Compare problems appear first in Grade 4, with the “times as much” language.	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?

Adapted from CCSS, p.89.

Table 3: The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a + b) + c = a + (b + c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) × c = a × (b × c)
Commutative property of multiplication	a × b = b × a
Multiplicative identity property 1	a × 1 = 1 × a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* × 1/*a* = 1/*a* × *a* = 1
Distributive property of multiplication over additions	a × (b + c) = a × b + a × c

Adapted from CCSS, p.90.

Table 4: The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c, then a = c.
Addition property of equality	If a = b, then a + c = b + c.
Subtraction property of equality	If a = b, then a – c = b – c.
Multiplication property of equality	If a = b, then a × c = b × c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Adapted from CCSS, p.90.