



## Grade 4 Mathematics – Evidence Statements

### Overview of the Maryland Comprehensive Assessment Program (MCAP)

The MCAP includes a coherent set of summative mathematics assessments aligned to the Maryland College and Career Ready Standards for Mathematics (MCCRSM). Students are required to take an MCAP mathematics assessment at the end of grades 3 – 8 and at the end of Algebra I. Students may also take an MCAP mathematics assessment at the end of Geometry and Algebra II.

The MCAP mathematics assessment development process is based on Evidence-Centered Design. The ECD process begins by establishing the answer to “What skills and understandings should be assessed?”. The MCCRSM describes the skills and understandings that the MCAP mathematics assessments assess. Assessments are then designed to gather evidence that allows inferences to be made. Assessments can be designed to allow inferences of various grain sizes. The MCAP mathematics assessments are summative assessments and are therefore designed to provide evidence that allows only general inferences about a student’s mathematical skills and understandings. The MCAP Mathematics Claims Structure describes the grain size of the evidence that the MCAP mathematics assessments will yield. Assessment items are designed to elicit evidence of a student’s level of proficiency for each claim.

#### MCAP MATHEMATICS CLAIMS STRUCTURE

##### Master Claim

The student is college and career ready or is on track to being college and career ready in mathematics.

##### Subclaims

**Content** - The student solves problems related to all content of the grade/course related to the Standards for Mathematical Practice.

**Reasoning** - The student expresses grade/course level appropriate mathematical reasoning.

**Modeling** - The student solves real-world problems with a degree of difficulty appropriate to the grade/course.

### MCAP MATHEMATICS ASSESSMENT ITEM TYPES

Item Type	Description	Subclaim	Scoring Method	Number of Operational Items per Form
<b>Type I</b>	Type I items will assess conceptual understanding, procedural skills, reasoning, and the ability to use mathematics to solve real-world problems.	<ul style="list-style-type: none"> <li>• Content</li> <li>• Reasoning</li> <li>• Modeling</li> </ul>	Machine scored	<b>31</b>
<b>Type II</b>	Type II items assess a student's ability to reason mathematically. Items may require students to provide arguments or justifications, critique the reasoning of others, and to use precision when explaining their thinking related to mathematics.	<ul style="list-style-type: none"> <li>• Reasoning</li> </ul>	Human scored	<b>2</b>
<b>Type III</b>	Type III items assess a student's ability to apply their understanding of mathematics when solving real-world contextual problems.	<ul style="list-style-type: none"> <li>• Modeling</li> </ul>	Human scored	<b>2</b>
			<b>Total</b>	<b>35</b>

## Overview of the MCAP Mathematics Evidence Statements

MCAP Mathematics Evidence Statements help teachers, curriculum developers, and administrators understand how the MCCRSM will be assessed. Assessment items are designed to elicit the evidence described in the Evidence Statements.

The MCAP Mathematics Evidence Statements for the Content Sub-Claim are organized using the same structure as the MCCRSM. The Domains, Clusters, and then Standards organize the Grade 4 Evidence Statements.

### Evidence Statements

Evidence statements are provided for each standard to describe the type of evidence that a task addressing the standard should elicit. In some cases, the standard clearly describes the type of evidence that an aligned task should elicit. The Evidence Statement for such standards will read “As stated in the standard”. In cases where the wording of a standard does not adequately describe the type of evidence that should be elicited, the Evidence Statement will attempt to better describe the type of evidence items should elicit. In cases where a standard is taught in both Algebra I and Algebra II, the Evidence Statement and/or Item Specification will seek to describe how the items might differ between the two courses.

## CODING OF CONTENT EVIDENCE STATEMENTS

Explanation of Coding	Example of the Evidence Statement
<p><b>Assessing the Entire Standard</b></p> <ul style="list-style-type: none"> <li>The evidence statement code is the same as the MCCRSM.</li> <li>The exact language and intent of the entire standard is assessed, which includes examples and “e.g.” parts of the standard.</li> </ul>	<p>3.OA.A.1</p> <p>Interpret products of whole numbers, e.g., interpret <math>5 \times 7</math> as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objectives can be expressed as <math>5 \times 7</math>.</p>
<p><b>Assessing Portions of a Standard with Multiple Operations</b></p> <ul style="list-style-type: none"> <li>The evidence statement code is the same as the MCCRSM with an addition of a dash and a sequential number, e.g. -1, -2, -3, ...</li> <li>The portion of the standard that is assessed will appear in bold font.</li> </ul>	<p>3.OA.A.3-1</p> <p><b>Use multiplication and division within 100 (both factors less than or equal to 10) to solve word problems in situations involving question groups, arrays, or area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</b></p> <p>3.OA.A.3-2</p> <p><b>Use multiplication and division within 100 (both factors less than or equal to 10) to solve word problems in situations involving question groups, arrays, or area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</b></p>

Explanation of Coding	Example of the Evidence Statement
<p><b>Assessing Portions of a Standard with Two or More Concepts</b></p> <ul style="list-style-type: none"> <li>The evidence statement code is the same as the MCCRSM with an addition of a dash and a sequential number, e.g. -1, -2, -3, ...</li> <li>The portion of the standard that is being assessed will appear in bold font.</li> </ul>	<p>4.OA.A.1-1</p> <p>Interpret a multiplication equation as comparison, e.g., interpret <math>35 = 5 \times 7</math> as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplication comparisons as multiplication equations.</p> <p>4.OA.A.1-2</p> <p>Interpret a multiplication equation as comparison, e.g., interpret <math>35 = 5 \times 7</math> as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. <b>Represent verbal statements of multiplication comparisons as multiplication equations.</b></p>

## CODING FOR REASONING EVIDENCE STATEMENTS

Explanation of Coding	Example of the Evidence Statement
<ul style="list-style-type: none"> <li>The evidence statement code begins with the corresponding grade level.</li> <li>The letter “R” appears after the grade level in the code to indicate Reasoning.</li> <li>The number at the end of the evidence statement code refers to a specific reasoning evidence statement.</li> </ul>	<p>4.R.1</p> <p>Base reasoning or explanations on a given pictorial representation and explain how the pictorial model represents a mathematical concept, or how it can be used to justify or refute a statement (with or without flaws) or how it can be used to generalize.</p>

**CODING FOR MODELING EVIDENCE STATEMENTS**

Explanation of Coding	Example of the Statement
<ul style="list-style-type: none"><li>• The evidence statement code begins with the corresponding grade level.</li><li>• The letter “M” appears after the grade level in the code to indicate Modeling.</li><li>• The number at the end of the evidence statement code refers to a specific modeling evidence statement.</li></ul>	4.M.1-1 Determine the problem that needs to be solved in a real-world situation.

## Standards for Mathematical Practice

The Standards for Mathematical Practice describe the varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These practice rest on important “processes and proficiencies” with longstanding importance in mathematics education.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Definitions

Defined below are some common terms used in the Evidence Statements.

- **Context:** The situation or setting for a word problem. The situations influence the solution path.
- **Thin Context:** A sentence or phrase that provides meaning for the quantity/quantities in a problem. For example, “The fractions represent lengths of a string.”
- **No context:** The item has no situation or setting. There are only numbers, symbols, and/or visual models in the item.
- **Visual models:** Drawn or pictorial examples that are representations of the mathematics.

## Content Subclaim

### 4.OA Operations and Algebraic Thinking

#### 4.OA.A Use the four operations with whole numbers to solve problems.

**4.OA.A.1** Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplication comparisons as multiplication equations.

**1-1.** Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.

**Evidence Statement:**

- 4.OA.A.1-1 focuses on interpreting a multiplication equation as multiplicative comparisons.

**Clarifications:**

- Type I items have thin to no context. See example in the standard.
- Type II and III items have context.
- Items include the wording “times as many as”.
- Items use multiplication expressions or equations up to  $10 \times 10$ .

**1-2.** Represent verbal statements of multiplication comparisons as multiplication equations.

**Evidence Statement:**

- 4.OA.A.1-2 focuses on representing verbal statements of multiplicative comparison as equations.

**Clarifications:**

- See 4.OA.A.1-1.
- Type II and III items have context that meets the language in the second sentence of the standard.

**4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

**Evidence Statement:**

- Focuses on solving word problems involving multiplicative comparison using the relationships between multiplication and division. It does not assess multiplication or division facts; it is to distinguish between multiplicative comparison and additive comparison.

**Clarifications:**

- Type Refer to the Multiplication and Division Situations table, found in the back of this document. Items sample equally from the situations in the third row of the table.
- Items may include bar models, tape diagrams, or other visual models that are appropriate for the content.

**4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders are interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

**3-1.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations. Represent these problems using equations with a letter standing for the unknown quantity.

**Evidence Statement:**

- 4.OA.A.3-1 focuses on solving multi-step word problems in which the remainder is **not** interpreted.

**Clarifications:**

- For content items, the last sentence in the standard is not assessed.
- Items include any of the four operations.
- Items include representing the problems using a variable for the unknown. Variables are lower case and italic font when typed.
- For addition and subtraction, note standards 4.NBT.4-1 and 4.NBT.4-2 for limitations on the number of digits allowed.
- For multiplication and division, note standards 4.NBT.B.5-1, 4.NBT.B.5-2, and 4.NBT.B.6 for limitation on the number of digits that can be used in problems.

**3-2.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, **including problems in which remainders are interpreted**. Represent these problems using equations with a letter standing for the unknown quantity.

**Evidence Statement:**

- 4.OA.A.3-2 focuses on solving multi-step word problems in which the remainder is interpreted.

**Clarifications:**

- See 4.OA.A.3-1.

**4.OA.B Gain familiarity with factors and multiples.**

**4.OA.B.4** Find all factor pairs for whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range of 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range of 1-100 is prime or composite. **Instructional standard only. The content of the standard is assessed in other standards.**

**4.OA.C Generate and analyze patterns.**

**4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1 generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items must provide the rule.
- Items do not require students to determine a rule.
- Items may involve extending patterns.
- Items may involve generalizing patterns.

**4.NBT Numbers and Operations in Base Ten**

Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

**4.NBT.A Generalize place value understanding for multi-digit whole numbers.**

**4.NBT.A.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Grade 4 expectations are limited to whole numbers less than or equal to 1,000,000.

**4.NBT.A.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $\geq$ ,  $=$ , and  $\leq$  symbols to record the results of comparisons.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items assess conceptual understanding, e.g., by including a mixture of expanded form, number names, and base ten numerals within items.
- Grade 4 expectations are limited to whole numbers less than or equal to 1,000,000.

**4.NBT.A.3** Use place value understanding to round multi-digit whole numbers to any place.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items have thin to no context.
- Grade 4 expectations are limited to whole numbers less than or equal to 1,000,000.

**4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.****4.NBT.B.4-1** Fluently add and subtract multi-digit whole numbers using the standard algorithm.**4-1.** Fluently add multi-digit whole numbers using the standard algorithm.**Evidence Statement:**

- 4.NBT.B.4-1 focuses on adding multi-digit whole numbers using the standard algorithm.

**Clarifications:**

- Items do not have context.
- Addends are limited to numbers with up to four-digits.
- The given addends are such as to require an efficient/standard algorithm (e.g.,  $7263 + 487$ ). Addends in the item do not suggest any obvious ad hoc or mental strategy (as would be present in a case such as  $1699 + 3501$ ).

**4-2.** Fluently subtract multi-digit whole numbers using the standard algorithm.**Evidence Statement:**

- 4.NBT.B.4-2 focuses on subtracting multi-digit whole numbers using the standard algorithm.

**Clarifications:**

- Items do not have context.
- Subtrahend and minuend may be three or four digits.
- The given subtrahend and minuend are such as to require an efficient/standard algorithm (e.g.,  $7263 - 4875$  or  $7406 - 4637$ ). The subtrahend and minuend do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as  $7300 - 6301$ ).

**4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply 2 two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.**5-1.** Multiply a whole number of up to four digits by a one-digit whole number using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Evidence Statement:**

- 4.NBT.B.5-1 focuses on multiplying a whole number of up to four-digits by a one-digit number.

**Clarifications:**

- Type I content items do not have context.
- Type II and III items may be written to include both bolded sentences in the standard.

**5-2.** Multiply two, two- digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Evidence Statement:**

- 4.NBT.B.5-2 focuses on multiplying two two-digit numbers.

**Clarifications:**

- See Clarifications for 4.NBT.B.5-2.

**4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items do not have context.
- Items are written to enable students to explain (show) the strategies used using words and/or equations. The intent of the standard is more than finding the correct answer.
- Items will require students to find whole number quotient using three- and four-digit dividends and one-digit divisors.

**4.NF Numbers and Operations – Fractions****4.NF.A Develop understanding of fractions as numbers.**

**4.NF.A.1** Explain why a fraction  $\frac{a}{b}$  is equivalent to a fraction  $\frac{n \times a}{n \times b}$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Items may include fractions that equal whole numbers less than or equal to 5.
- Items may include visual fraction models such as bar models/tape diagrams, number lines, or area models.

**4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbol  $\geq$ ,  $=$ ,  $\leq$ , and justify the conclusions, e.g., by using a visual fraction model.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items include fractions that elicit the use of benchmark fractions to compare.
- Items include language to emphasize that comparisons are valid only when the two fractions refer to the same whole.
- Items are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Items may include fractions that equal whole numbers less than or equal to 5.
- Items may include visual fraction models such as bar models/tape diagrams, number lines, or area models.

**4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

**4.NF.B.3** Understand a fraction  $\frac{a}{b}$  with  $a \geq 1$  as a sum of fractions  $\frac{1}{b}$

**3a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

**Instructional standard only. The content of the standard is assessed in other standards.**

**3b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition as an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$  or  $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$  or  $2\frac{1}{8} = 1 + 1 + \frac{1}{8}$  or

$$2\frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$$

**Evidence Statement:**

- This standard addresses the language of the entire standard keeping in mind the essential understandings given in the first row for 4.NF.B.3a.

**Clarifications:**

- For Type I items, only the answer is required.
- Items are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Items may include fractions that equal whole numbers less than or equal to 5.
- Items may include visual fraction models such as bar models/tape diagrams, number lines, or area models.

**3c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

**Evidence Statement:**

- This standard addresses the language of the entire standard keeping in mind the essential understandings given in the first row for 4.NF.B.3a.

**Clarifications:**

- Items do not have context.
- Items are limited to fractions with denominators 2, 3, 4, 6, and 8. Use of grade limitations minimizes computational difficulty.
- Items include the examples given for this standard in the “e.g.,” portion of the standard.

**3d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

**Evidence Statement:**

- This standard addresses the language of the entire standard keeping in mind the essential understandings given in the first row for 4.NF.B.3a.

**Clarifications:**

- Items are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Items may include mixed numbers with like denominators.
- Refer to the *Addition and Subtraction Situations* table, found in the back of this document. Items that include these problem types are samples equally.
- Items may include visual fraction models such as bar models/tape diagrams, number lines, or area models.

**4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

**4a.** Understand a fraction  $\frac{a}{b}$  as a multiple of  $\frac{1}{b}$ . For example, use a visual fraction model to represent  $\frac{5}{4}$  as the product of  $5 \times \frac{1}{4}$  recording the conclusion by the equation  $\frac{5}{4} = 5 \times \frac{1}{4}$ .

**Evidence Statement:**

- The language in 4.NF.B.4 should be considered when developing assessment items. 4.NF.B.4a focuses on multiplying a whole number times a unit fraction.

**Clarifications:**

- Items have thin context.
- Items are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Items may include visual fraction models such as bar models/tape diagrams, number lines, or area models.

**4b.** Understand a multiple of  $\frac{a}{b}$  as a multiple of  $\frac{1}{b}$ , and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express  $3 \times \frac{2}{5}$  as  $6 \times \frac{1}{5} = \frac{6}{5}$ . (In general,  $n \times \frac{a}{b} = \frac{(n \times a)}{b}$ .)*

**Evidence Statement:**

- The language in 4.NF.B.4 should be considered when developing assessment items. 4.NF.B.4b focuses on multiplying a whole number times a unit fraction.

**Clarifications:**

- Items have thin context.
- Items are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Items may include visual fraction models such as bar models/tape diagrams, number lines, or area models.

**4c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $\frac{3}{8}$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

**Evidence Statement:**

- The language in 4.NF.B.4 should be considered when developing assessment items. 4.NF.B.4c focuses on multiplying a whole number times a fraction.

**Clarifications:**

- Items have thin context.
- Items are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Situations are limited to those in which the product is unknown. Situations do not include unknown factors.
- Results may equal fractions greater than 1, including fractions equal to whole numbers less than or equal to 5.
- Items may include visual fraction models such as bar models/tape diagrams, number lines, or area models.

**4.NF.C Understand decimal notation for fractions, and compare decimal fractions.**

**4.NF.C.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. For example, when adding  $\frac{3}{10}$  as  $\frac{4}{100}$  express  $\frac{3}{10}$  as  $\frac{30}{100}$ , then  $\frac{30}{100} + \frac{4}{100} = \frac{34}{100}$ .

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items may have thin to no context.

**4.NF.C.6** Use decimal notation for fractions with denominators 10 and 100. For example, rewrite 0.62 as  $\frac{62}{100}$  or describe a length as 0.62 meters or locate 0.62 on a number line diagram.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Understand that 0.62 and  $\frac{62}{100}$  are different representations for the same number.
- See the example provided in the standard when using meters or centimeters on a number line when measuring length.

**4.NF.C.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ ,  $<$ , and justify the conclusions, e.g., by using a visual model.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Type I items have thin context.
- Items may require students to record the results using comparison symbols.
- Items may ask students to order decimals from least to greatest or greatest to least.

**4.MD Measurement and Data****4.MD.A Solve problems involving measurement and conversion of measurements for a larger unit to a smaller unit.**

**4.MD.A.1** Know relative sizes of measurement units within one system of units including km, m, cm, kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12),(2, 24), 3, 36), etc.*

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items have context or thin context.
- Measurement units should be within one system of units, not across systems.
- Items only include conversion from larger units in terms of smaller units.

**4.MD.A.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems that require simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Grade 4 expectations are limited to whole numbers less than or equal to 1,000,000.
- Items with fractions are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

**4.MD.A.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formulas as a multiplication equation with an unknown factor.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items. The intent of the standard is to apply the area and perimeter formulas for rectangles and the relationship between the two concepts.

**Clarifications:**

- Items must use grade 4 appropriate numbers (not all single digit values). Numbers must be reasonable so as not to impede finding the solution.

**4.MD.B Represent and interpret data.**

**4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Type II and Type III items will include both sentences.
- Items are limited to fractions with denominators of 2, 4, or 8.
- Items may include mixed numbers with stated denominators.
- Items may include fractions equivalent to whole numbers equal to or less than 5.
- Items do not require computations beyond the grade 4 expectations.
- Line plot data points are represented by “X” rather than dots.

**4.MD.C Geometric Measurement: Understand concepts of angle and measure angles.**

**4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

**5a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through  $\frac{1}{360}$  of the circle is called a “one-degree angle,” and can be used to measure angles.

**Instructional standard only. The content of this standard is assessed in other standards.**

**5b.** An angle that turns through  $n$  one-degree angles is said to have an angle measure of  $n$  degrees.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Consider content of 4.MD.C.5a when developing items for 4.MD.C.5b.

**4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Consider content of 4.MD.C.5a and 4.MD.C.5b when developing items for 4.MD.C.6.

**4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- N/A

**4.G Geometry****4.G.A Draw and identify lines and angles, and classify shape by properties of their lines and angles.**

**4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- N/A

**4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- A trapezoid is defined as “A quadrilateral with at least one pair of parallel sides.”
- Students will not be asked to provide a definition of the word, trapezoid. They should understand the definition, so they are able to recognize attributes of the trapezoid.
- Items may include terminology: equilateral, isosceles, scalene, acute, right, and obtuse.
- When identifying right triangles, the 90-degree symbol may be used or the language, “appear to be right angles” may be used.

**4.G.A.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

**Evidence Statement:**

- The language of the standard guides the creation of assessment items.

**Clarifications:**

- Items assess symmetry only with shapes (not letters, e.g.)
- When asking for the number of lines of symmetry on a composite shape, do not include interior lines that show the individual shapes that make up the composite shape.

## Reasoning Subclaim

All reasoning assessment items connect to both the Grade 4 reasoning evidence statements and the content evidence statements. Students must provide evidence of their ability to reason mathematically by responding to Type I and Type II items.

### Type I

- Items are machine scored.
- Items are 1 point per item.
- Items may be aligned to any of the content standards.
- In Grade 3, items include one step. In Grades 4 and 5, items include one or two steps.\*
- Calculators are allowed on all reasoning items.
- Four items from this grouping will appear on each assessment.

### Type II

- Items are human scored constructed response.
- In Grades 3 and 4, items are 3 points per item. In Grade 5, items are 3 points or 4 points per item.
- Items may be aligned to any of the content standards.
- In Grade 3, items include two steps. In Grades 4 and 5, items include at least three steps.\*
- Calculators are allowed on all reasoning items.
- Two items from this grouping will appear on each assessment.

\* The number of steps is determined by the number of different operations (if an operation repeats in another part of the solution, it only counts as one step), interpretation of a visual model, a chart or table, a graph, or a remainder.

The following pages provide the Reasoning Evidence Statements, specific clarifications, as well as sample items. Please keep in mind that sample items are used to help clarify the evidence statement. They are not a template for assessment items.

**4.R.1 Evidence Statement:**

- Base reasoning or explanations on a given pictorial representation and explain how the pictorial model represents a mathematical concept, or how it can be used to justify or refute a statement (with or without flaws) or how it can be used to generalize.

*Type I items are 1-point machine scored. Grade 3 items include one step. Grade 4 and 5 items include one or two step(s).*

**Clarifications:**

- Items have a mathematical or real-world context.
- Pictorial representations are a representation of a specific mathematics concept. Visuals that provide information such as tables, charts, graphs, calibrated containers, etc. are not appropriate for this evidence statement.
- Items may provide a pictorial representation for students to identify the mathematical concept the representation exemplifies. Students select the mathematics represented by the pictorial representation (as shown in Appendix: Example 1).
- Items may provide a mathematical concept and require students to select the correct corresponding pictorial representation. Students select the pictorial representation that best explains a given mathematical concept or procedure (as shown in Appendix: Example 2).
- Items may state a conjecture or generalization and students select the pictorial representation that proves or disproves the conjecture or generalization. Students select the pictorial representation that proves or disproves a given conjecture or generalization (as shown in Appendix: Example 3).

*Type II items are 3-point human scored. Grade 3 items include two steps. Grade 4 and 5 items include at least 3 steps.*

**Clarifications:**

- Items have a rich mathematical or real-world context. The context supports a problem that requires two or more steps to solve.
- Pictorial representations are a representation of a specific mathematics concept. Visuals that provide information such as tables, charts, graphs, calibrated containers, etc. are not appropriate for this evidence statement.
- Items allow students to provide work and/or a written explanation and/or to use the drawing tool to describe their own reasoning.
- Items may provide a pictorial representation for students to explain how the pictorial model represents the mathematics (as shown in Appendix: Example 4).
- Items may state a conjecture or generalization and students explain why the pictorial representation supports or does not support the conjecture or generalization (as shown in Appendix: Example 5).

**4.R.2 Evidence Statement:**

- Identify flawed thinking or reasoning and explain how to correct the thinking or work.

*Type I items are 1-point machine scored. Grade 3 items include one step. Grade 4 and 5 items include one or two step(s).*

**Clarifications:**

- Items have a rich mathematical or real-world context. The context supports a problem that requires two or more steps to solve.
- Pictorial representations are a representation of a specific mathematics concept. Visuals that provide information such as tables, charts, graphs, calibrated containers, etc. are not appropriate for this evidence statement.
- Items allow students to provide work and/or a written explanation and/or to use the drawing tool to describe their own reasoning.
- Items may provide a pictorial representation for students to explain how the pictorial model represents the mathematics (as shown in Appendix: Example 6).
- Items may state a conjecture or generalization and students explain why the pictorial representation supports or does not support the conjecture or generalization (as shown in Appendix: Example 7).

*Type II items are 3-point human scored. Grade 3 items include two steps. Grades 4 and 5 items include at least 3 steps.*

**Clarifications:**

- Items have a rich mathematical or real-world context. The context supports a problem that requires two or more steps to solve.
- Items provide only incorrect work/thinking. Items do not ask students to identify if work is correct or incorrect.
- Items allow students to provide work and/or a written explanation and/or to use the drawing tool to describe their own reasoning. Students may use multiple representations to support or further explain their own reasoning.
- Items may prompt students to identify the flaw AND explain how to correct the flaw (as shown in Appendix: Example 8).
- Items may prompt students to identify the flaw AND to correctly solve the problem (as shown in Appendix: Example 9).

**4.R.3 Evidence Statement:**

- Prove or disprove a statement, conjecture, or generalization, using correct and precise mathematical examples (visual representations, words, symbols, equations, or expressions.)

*Type I items are 1-point machine scored. Grade 3 items include one step. Grades 4 and 5 items include one or two step(s).*

**Clarifications:**

- Items have a mathematical or real-world context.
- Items state a conjecture based on grade appropriate mathematical concept (number sense, fraction concepts, properties of operations, relationship between operations, place values, etc.) that is either true or false.
- Items require students to either support the conjecture or show that it is not true using mathematical examples (as shown in Appendix: Example 10). Items do not require students to identify if the conjecture is correct or incorrect.
- Items may include visual representations, expressions, or equations.

*Type II items are 3-point human scored. Grade 3 Items include two steps. Grades 4 and 5 items include at least 3 steps.*

**Clarifications:**

- Items have a rich mathematical or real-world context. The context supports a problem that requires two or more steps to solve.
- Items state a conjecture based on grade-appropriate mathematical concept (number sense, fraction concepts, properties of operations, relationship between operations, place values, etc.) that is either true or false.
- Items require students to provide their own mathematical examples to support the conjecture or to show that the conjecture is not true (as shown in Appendix: Examples 11 and 12). Items do not require students to identify if the conjecture is correct or incorrect.
- Items may include visual representations, equations, and expressions.

**4.R.4 Evidence Statement:**

- Reason mathematically to create or analyze a correct and precise solution to a real-world problem and be able to explain why the answer is mathematically correct.

*Type I items are 1-point machine scored. Grade 3 items include one step. Grades 4 and 5 items include one or two step(s).*

**Clarifications:**

- Items have a mathematical or real-world context.
- Item context or answer choices may be based on big mathematics concepts, such as understanding of place value concepts, number concepts, properties, or relationships among whole numbers, fractions and decimals, etc.
- Items may prompt students to select an explanation why a procedure or strategy is correct or appropriate for solving a given problem (as shown in Appendix: Example 13).
- Items may prompt students to select a strategy or procedure that is best used to solve a given problem (as shown in Appendix: Example 14).

*In Grades 3 and 4, Type II items are 3-point human scored. In Grade 5, Type II items are 4-point human scored. Grade 3 items include two steps. Grade 4 and 5 items include at least three steps.*

**Clarifications:**

- Items have a rich mathematical or real-world context. The context supports a problem that requires two or more steps to solve.
- Item context may be based on big mathematics concepts, such as understanding of place value concepts, number concepts, properties, or relationships among whole numbers, fractions and decimals, etc.
- Items may prompt students to explain why a solution or procedure is mathematically correct or why the answer makes sense (as shown in Appendix: Example 15).
- Items may prompt students to provide or explain a valid chain of reasoning that results in a given solution to a problem (as shown in Appendix: Example 16).

## Modeling Subclaim

All modeling assessment items connect to both the Grade 3 modeling evidence statements and the content evidence statements. Students must provide evidence of their ability to apply one or more steps of the modeling cycle by responding to Type I and Type III items.

### Type I

- Items are aligned to M.1-1, M.1-2, or M.1-3.
- Items are machine scored.
- Items are 1 point per item.
- Items may be aligned to any of the content standards.
- In Grade 3, items include one step. In Grades 4 and 5, items include one or two steps.\*
- Calculators are provided on all modeling items.
- Four items from this grouping will appear on each assessment.

### Type III

- Items are aligned to M.1-4 or M.1-5.
- Items are human scored constructed response.
- In Grades 3 and 4, items are 3 points per item. In Grade 5, items are 3 points or 4 points per item.
- Items may be aligned to any of the content standards.
- In Grade 3, items include two steps. In Grades 4 and 5, items include at least three steps.\*
- Calculators are allowed on all modeling items.
- Two items from this grouping will appear on each assessment.

\* The number of steps is determined by the number of different operations (if an operation repeats in another part of the solution, it only counts as one step), interpretation of a visual model, a chart or table, a graph, or a remainder.

Modeling items can have context even if the aligned content evidence statement clarifies that “Items do not have context”.

The following pages provide the modeling evidence statements, specific clarifications, as well as sample items. Please keep in mind that sample items are used to help clarify the evidence statement. They are not a template for assessment items.

*Type I items are 1-point machine scored. Grade 3 items include one step. Grade 4 and 5 items include one or two step(s).*

**4.M.1-1 Evidence Statement:**

- Determine the problem that needs to be solved in a real-world situation.

**Clarifications:**

- Items have a rich real-world context.
- Items do not require a solution.
- Items may include charts or graphs that could be analyzed for information about the problem.
- Items may require students to identify the problem that needs to be solved (as shown in Appendix: Example 17).
- The context of the problem may be a numberless word problem. This allows students to focus on the context of the problem, not just the numbers (as shown in Appendix: Example 18).

**4.M.1-2 Evidence Statement:**

- Determine the information that is needed to solve a problem in a given real-world situation.

**Clarifications:**

- Items have a rich real-world context.
- Items do not require a solution.
- Items may include charts or graphs that can be analyzed for information.
- Items may prompt students to identify the information, from a given problem, that is needed to solve the problem (as shown in Appendix: Example 19).
- Items may not provide all of the information needed to solve the problem. Students will make conclusions based on the information that is given in the problem.

**4.M.1-3 Evidence Statement:**

- Identify the mathematics that is needed to create a solution path for a real-world situation.

**Clarifications:**

- Items have a rich real-world context.
- Items do not require a solution.
- Items may prompt the students to identify the sequence of operations needed to create a solution path. The numbers used in the problem are not required in answer choices (as shown in Appendix: Example 20).
- Items may prompt students to identify an expression with the correct sequence of operations, write an equation with a letter for the answer, or to write expressions.
- Answer choices should be mathematically correct and precise.

*In Grades 3 and 4, Type III items are 3-point human scored. In Grade 5, Type III items are 3- or 4-point human scored. Grade 3 items include two steps. Grade 4 and 5 items include at least three steps.*

**4.M.1-4 Evidence Statement:**

- Create a solution path that represents the mathematics needed to solve a real-world situation.

**Clarifications:**

- Items have a rich real-world context that supports a problem that requires two or more steps to solve.
- Items require one or more complete and accurate solution paths that include the answer (as shown in Appendix: Example 21).

**4.M.1-5 Evidence Statement:**

- Evaluate a partial or complete solution to a real-world situation.

**Clarifications:**

- Items have a rich real-world context that supports a problem that requires two or more steps to solve.
- Items require students to analyze a given solution path (partial or complete) to determine if it represents mathematically correct thinking for the given real-world situation. Students should analyze and explain how the solution path represents the problem (as shown in Appendix: Example 22).

## Addition and Subtraction Situations

	Results Unknown	Change Unknown	Start Unknown
<b>Add to</b>	<p>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?</p> $2 + 3 = ?$	<p>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two bunnies?</p> $2 + ? = 5$	<p>Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?</p> $? + 3 = 5$
<b>Take from</b>	<p>Five apples were on the table. I ate two apples. How many apples are on the table now?</p> $5 - 2 = ?$	<p>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?</p> $5 - ? = 3$	<p>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?</p> $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown
<b>Put Together/ Take Apart</b>	<p>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?</p> $2 + 3 = ?$	<p>Five apples are on the table. Three are red and the rest are green. How many apples are green?</p> $3 + ? = 5$ $5 - 3 = ?$	<p>Grandma has five flowers. How many can she put in the red vase and how many in her blue vase?</p> $5 = 0 + 5$ $5 = 5 + 0$ $5 = 1 + 4$ $5 = 4 + 1$ $5 = 2 + 3$ $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	<p><b>How many more?" version:</b></p> <p>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?</p>	<p><b>Version with "more":</b></p> <p>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?</p>	<p><b>Version with "more"</b></p> <p>Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?</p>
	<p><b>"How many fewer?" version:</b></p> <p>Lucy has two apples. Julies has five apples. How many fewer apples does Lucy have than Julie?</p> <p style="text-align: center;"><math>2 + ? = 5</math></p> <p style="text-align: center;"><math>5 - 2 = ?</math></p>	<p><b>Version with "fewer":</b></p> <p>Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have?</p> <p style="text-align: center;"><math>2 + 3 = ?</math></p> <p style="text-align: center;"><math>3 + 2 = ?</math></p>	<p><b>Version with "fewer"</b></p> <p>Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?</p> <p style="text-align: center;"><math>5 - 3 = ?</math></p> <p style="text-align: center;"><math>? + 3 = 5</math></p>

## Multiplication and Division Situations

Problem Situation	Unknown Product	Group Size Unknown (How many in each group?) $3 \times ? = 18$ and $18 \div 3 = ?$	Number of Groups Unknown (How many groups?)
<p><b>Equal groups</b> (Grades 3 – 5)</p>	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p>Measurement example: Alex needs 3 lengths of string, each 6 inches long. How much string will Alex need?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p>Measurement example: Dale has 18 inches of string, which he will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p>Measurement example: Ruth has 18 inches of string, which she will cut into pieces that are 6 inches long. How many pieces of string will she have?</p>
<p><b>Arrays and Area</b> (Grades 3 – 5)</p>	<p>There are 3 rows of apples with 6 in each row. How many apples are there? OR</p> <p>The apples in the grocery window are in 3 rows and 6 columns. How many apples are there?</p> <p>Area Example: What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row? OR</p> <p>If 18 apples are arranged into an array with 3 rows, how many columns of apples are there?</p> <p>Area example: A rectangle has an area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? OR</p> <p>If 18 apples are arranged into an array with 3 columns, how many rows are there?</p> <p>Area example: A rectangle has an area of 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>

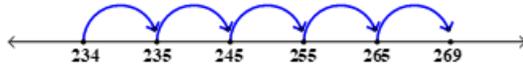
Problem Situation	Unknown Product	Group Size Unknown (How many in each group?) $3 \times ? = 18$ and $18 \div 3 = ?$	Number of Groups Unknown (How many groups?)
<p style="text-align: center;"><b>Compare</b> <b>(Grades 4 – 5)</b></p> <p>Multiplicative Compare problems appear first in Grade 4, with the “times as much” language.</p>	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p>Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p>Measurement example: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p>Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>

# Appendix

## Sample Reasoning Items

### Example 1

A student uses the number line to find the value of  $269 - 234$ .

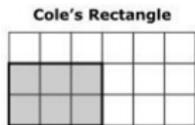


Which expression represents how the student used the number line to find the value of  $269 - 234$ ?

- A.  $10+10+10+10+10+10+10+10+10+10$
- B.  $10+10+10+10+11+10+10+10+1$
- C.  $4+10+10+10+14+10+10+10+1$
- D.  $1+10+10+10+11+10+10+10+1$

### Example 2

Cole drew the shaded rectangle on a piece of grid paper.



Nora also drew a rectangle on grid paper. Nora's rectangle has the same perimeter as Cole's rectangle but has a different area than Cole's rectangle. Which rectangle could be Nora's?

- A. **Nora's Rectangle**
- B. **Nora's Rectangle**
- C. **Nora's Rectangle**
- D. **Nora's Rectangle**

**Example 3**

A student says that some quadrilaterals are not rhombuses.

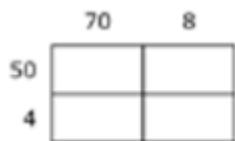
Which **three** figures prove that the student's statement is true?

Select the **three** correct answers.

**Example 4**

A model is shown.

Explain how the model could be used to find the result of  $54 \times 78$ . Then find the result of  $54 \times 78$ .



Enter your answer and your explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

**Example 5**

A friend uses this model to show one whole.



The friend drew a model to represent the product  $\frac{1}{3} \times 9$



The friend thinks that this model can represent the multiplication expression. Explain how the model disproves the friend's thinking. Enter your answer and your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

**Example 6**

An artist drew a rectangle.

The artist said the area of the rectangle shown is found by adding  $3 + 7 + 3 + 7 = 20$ . The artist made a mistake.

Which statement explains the artist's mistake?

- A. The artist used the incorrect side lengths.
- B. The artist added the side lengths incorrectly.
- C. The artist calculated the final answer incorrectly.
- D. The artist calculated the perimeter of the rectangle.

**Example 7**

A student said the value of 11 tens, 8 ones and 2 hundreds is 1182.

The student made a mistake.

What is the correct value?

Enter your answer in the space provided.

**Example 8**

Keisha said the value of 11 tens, 8 ones and 2 hundreds is 1182.

What error did the student make in her reasoning?

Explain how you would correct the error that the student made.

Enter your answer and your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

**Example 9**

An explanation for finding the value of  $4 \times 2 \times 3$  is given:

- Multiply  $4 \times 2$ .
- Multiply  $4 \times 3$ .
- Add the two products.

A mistake is made in the explanation. Describe the mistake and explain a strategy that could be used to find the product for  $4 \times 2 \times 3$ .

Enter your answer and your work or explanation in the space provided.

You may also use the drawing tool to help explain or support your answer.

**Example 10**

A student says that two fractions can be compared using the benchmark fraction  $\frac{1}{2}$ .

Which **two** comparisons prove the students' thinking is correct?

Select the **two** correct answers.

A.  $\frac{7}{8} \geq \frac{3}{12}$

B.  $\frac{1}{4} \leq \frac{4}{8}$

C.  $\frac{2}{6} \leq \frac{5}{6}$

D.  $\frac{6}{10} \leq \frac{6}{8}$

E.  $\frac{6}{12} \geq \frac{2}{8}$

**Example 11**

Your teacher gives you this problem to solve.

*What happens to the sum in an addition problem if each addend is multiplied by two?*

What is the answer to the problem the teacher gave? Explain how you found the answer and provide two examples that support your answer.

Enter your answer and your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

**Example 12**

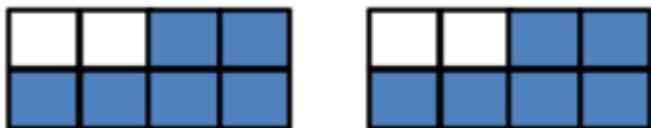
In a third grade classroom, the teacher asked two students to name a fraction. Student A says  $\frac{5}{8}$  and Student B says  $\frac{3}{4}$ . The teacher asks, if both fractions have the same size whole, how are  $\frac{5}{8}$  and  $\frac{3}{4}$  alike and how are they different?

Explain how the two fractions are alike and different and include two examples to explain your thinking.

Enter your answer and your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

**Example 13**

A pair of fraction models is shown.



Which statement explains the correct reasoning for the sum of the shaded parts?

- A. Each model shows  $\frac{5}{8}$  shaded, so the sum is  $\frac{26}{8}$ .
- B. Each model shows  $\frac{6}{8}$  shaded, so the sum is  $2\frac{6}{8}$ .
- C. Each model is divided into 8 parts. There are 12 total parts shaded, so the sum is  $\frac{12}{8}$ .
- D. Each model is divided into 8 parts. There are 12 total parts shaded, so the sum is  $1\frac{2}{8}$ .

**Example 14**

Which strategy can be used to multiply  $8 \times 7$ ?

- A. Multiply  $8 \times 2$  and then add 7.
- B. Multiply  $10 \times 7$  and then subtract 2.
- C. Multiply  $4 \times 2$  three times and add  $8 + 8 + 8$ .
- D. Multiply  $2 \times 7$  four times and add  $14 + 14 + 14 + 14$ .

**Example 15**

A contractor measured the length and width of two rectangular pieces of land.

- The two pieces of land are adjacent and share the same width of 17 yards.
- The first piece of land has a length of  $32\frac{1}{3}$  yards.
- The second piece of land has a length of  $25\frac{1}{4}$  yards.

Find the total area, in square yards, of both pieces of land.

Analyze your work. Explain how your work correctly represents the problem.

Enter your answer, your work and your explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

**Example 16**

A group of four shapes is shown.



- Pick two shapes from the group and explain why each shape does not belong in the group.
- Think about another shape that you could add to the group. Explain why that shape does not belong to the group.

Enter your answer, your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

## Sample Modeling Items

### Example 17

Two science classes are conducting an experiment together in the science lab. Each class has 23 students. The tables in the science lab can each seat up to 4 students.

Which question could be answered using **all** the information in the problem?

- A. How many students enjoy the science class?
- B. How many science books are needed for one class?
- C. How many students can be seated at all of the tables?
- D. How many tables are needed for all the students from both classes?

### Example 18

Some science classes are conducting an experiment together in the science lab. Each class has the same number of students. Each table can seat the same number of students.

Which question could be answered using **all** the information in the problem?

- A. How many tables are needed for one class?
- B. How many students are there in all of the classes?
- C. How many science books are needed for one class?
- D. How many tables are needed for all the students from both classes?

**Example 19**

Two science classes are conducting an experiment together in the science lab. Each class has 23 students. The tables in the science lab can each seat up to 4 students. How many tables are needed for all the students from both classes?

Which three pieces of information are needed to solve the problem?

Select the **three** correct answers.

- A. Each class has 23 students.
- B. Each table seats up to 4 students.
- C. Two science classes are in the lab.
- D. The experiment is in the science lab.
- E. Classes are conducting an experiment.
- F. The students work on the experiment for 30 minutes.

**Example 20**

Two science classes are conducting an experiment together in the science lab. Each class has 23 students. The tables in the science lab can each seat up to 4 students.

How many tables are needed for all the students from both classes?

Which operations could be used to solve the problem?

- A. only division
- B. only multiplication
- C. first division and then addition
- D. first addition and then division

**Example 21**

Two science classes are conducting an experiment together in the science lab. Each class has 23 students. The tables in the science lab can each seat up to 4 students.

How many tables are needed for all the students from both classes?

Write an equation or equations to show how to find the least number of tables needed for all the students from both classes in the science lab.

Enter your answer and your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

**Example 22**

Two science classes are conducting an experiment together in the science lab. Each class has 23 students. The tables in the science lab can each seat up to 4 students. How many tables are needed for all the students from both classes?

One student's work and explanation for the problem is given:

$$23 \times 2 = 46$$

$$46 \div 4 = t$$

*t = 11 with remainder of 2 (total number of tables)*

*12 tables are needed.*

Analyze the student's work. Explain how the student's work correctly or incorrectly represents the problem.

Enter your answer and your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.