Transcript for MSDE - MCAP Mathematical Modeling in High School

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Hello and welcome to the Maryland State Department of Education's presentation of modeling in high school mathematics. My name is Michael Turlik and I am a mathematics assessment specialist at MSDE.

The objectives of this presentation are as follows. Participants will investigate what is meant by modeling in high school mathematics and understand how modeling is assessed on the MCAP mathematics assessment.

What is mathematical modeling in the K-12 Classroom? A quick interpretation is as follows: **Overview** Mathematical models describe situations in the world. Albert Einstein wondered "*How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably adapted to the objects of reality?"*

Both that overview and the quote from Einstein come from the modeling progression document from common core. This is a fantastic tool and resource to help explain how modeling is developed throughout a student's educational progression.

On Page 7 of this document has something to say in regards to modeling. One of the practice standards—Model with mathematics (MP.4)— focuses on modeling, and modeling draws on and develops all eight. This helps explain why modeling with mathematics and statistics is challenging. It is a capstone, the proof of the pudding. To embody it, students might complete a capstone experience in modeling.

Mathematical Modeling plays a large role on our state assessment. As a state, we give assessments to meet federal guidelines for accountability for our students. Our master claim we are trying to assess with our mathematics assessments is if student's are on-track to being college and career ready in mathematics. How we can prove that they are is by having three subclaims that are reported out when students take one of our assessments. The content subclaim asks students to solve problems related to the grade or course level content. The reasoning subclaim asks students to express grade level appropriate mathematical reasoning and the modeling Subclaim asks students to solve real-world problems with a degree of difficulty appropriate to the course. If we take a deeper dive into our subcategories at the high school level, you can see how each of the categories are weighted. All of the assessments for the high school level are approximately 48 points, where the content category represents about half of those points and reasoning and modeling both make up a fourth of those points.

On the MCAP high school mathematics assessments, our items are broken up into 3 types, type I items which are 1 point machine scored items, and Type II and III items which represent modeling and reasoning Constructed response items that are 4 points and are human scored. Items that assess student's ability to model with mathematics are found in both type I and Type III items. New for the state of Maryland, is that modeling is now assessed with 1 point items. This is positive news in two ways, one, it allows students to show their ability to model with mathematics in a more accessible way as one point items generally take less time to complete. Secondly it allows for reporting on the modeling Subclaim to be based off of both machine scored and human scored items.

Modeling items on the MCAP assessment are written based off our evidence statements. These evidence statements can be found on our website and exist for each of the three end of course exams. An example of the modeling section of the evidence statements is shown below. The code for the statement is listed on the left hand side. In A1.M.1 the A1 stands for algebra I, the M for modeling and the 1 for the order of the evidence statements. G is used for geometry and A2 is used for algebra II. The next column shows the modeling evidence statement and the last column shows clarifications or limits as to what can be assessed for that evidence statement.

All items that assess modeling evidence statements also need to be aligned to at least one content evidence statement in the same course. The content evidence statement is considered an "Additional Standard" Type I and Type III items may be written for each modeling evidence statement unless stated in the evidence statement. Type I items are machine scored, are 1 point per item, and can be applied to any of the content standards unless otherwise noted in the evidence statements. All modeling items are calculator active, meaning that a student will always have a calculator when these items appear on an assessment. Four items from this grouping will appear on each assessment. Type III items are constructed response reasoning items. They are 4 points per item, can be applied to any of the content standards, unless otherwise noted on the evidence statement documents. Like the type I modeling items, all Type III items are calculator active and two of these items will appear on each high school assessment.

What does it mean to model with mathematics? Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Some resources that can help foster modeling in the classroom, the first is the common core state standards introduction to modeling. Modeling is defined in this document as ”the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions”

Further the modeling standards are ***best interpreted not as a collection of isolated topics but rather in relation to other standards.*** Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol.

Many of the resources that talk about modeling with mathematics focus on the modeling cycle. In this cycle students are giving a problem and asked to report a solution. In order to get to that solution students have to formulate, compute, interpret and validate. This is not always a linear process and can be done in multiple different orders. We will talk more in depth about the stages of the modeling cycle a little later in the presentation.

To get more information on modeling with mathematics and how to assess modeling in your classroom there are many resources that you can use. Number 1 are the MCAP practice tests provided on the support.mdassessments.com. Beginning in 2021 the practice tests for each grade level are going to be more robust and have many more examples of modeling items.

Another great resource that explain the modeling process is the GAIMME report. Although it is rather long, it is broken up in to grade levels so that it is easy to read. One quote we pulled out from the report that helped inform our thinking on modeling is that “**Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.”**

Further the Gaimme report goes on to describe mathematical modeling: Most definitions of modeling we find emphasize this most important aspect, namely the relation between modeling and the world around us. Using the language of mathematics to quantify real-world phenomena and analyze behavior. Using math to explore and develop our understanding of real world problems. An iterative problem solving process in which mathematics is used to investigate and develop deeper understanding.

Another great resource for mathematical modeling is the SIAM document on math modeling. **Page 4** Modeling problems are entirely different than the types of word problems most of us encountered in math classes. In order to understand the difference between math modeling and word problems, consider the following questions about recycling.

1. The population of Yourtown is 20,000, and 35% of its citizens recycle their plastic water bottles. If each person uses 9 water bottles per week, how many bottles are recycled each week in Yourtown?
2. How much plastic is recycled in Yourtown?

Other resources that you can use to research modeling with mathematics and how modeling is assessed are the New Meridan Released items, the smarter balanced released items (which include examples of 1 point modeling items.)

The modeling cycle is the basis for most of the assessment items created for MCAP. With our 1-point items, we try to isolate a single stage of the modeling cycle and in our 4-point items, we try to aim for students completing more than one of the stages in their responses.

Lets take a deeper look at the formulate stage of the modeling cycle. *In this stage, students would* Make assumptions, Define variables and Create a mathematical models. Standards that align to this stage are A1.M.2 Construct a mathematical model to solve a problem, A1.M.7 Identify information or an assumption needed to solve a problem, and A1.M.8 Provide a reasoned estimate of a quantity needed to solve a problem.

Formulate stage, making assumptions. To build a mathematical model, students must **make assumptions.** Once they understand the real-world situation, the students brainstorm about which factors are important, which can be ignored, and what needed information is missing. This brainstorming leads students to make informed assumptions concerning the major factors that affect the real-world situation.

Let's look at a 1 point modeling item that showcases the formulate stage. This aligns to A1.M.7 which states to identify information or an assumption needed to solve a problem. An office manager is ordering this year's supply of pencils and pens for the office. Each pencil costs 10 cents and each pen costs 75 cents and the manager will spend a total of $235 on pencils and pens. Which additional piece of information is sufficient to determine the number of pencils and pens that the manager will order? A.) the number of pencils purchased last year will be equal to the number of pens purchased this year. B.) The number of pens purchased this year will be three times the number of pencils purchased this year. C.) the total cost of the pens purchased this year will be greater than the total cost of the pencils purchased this year. D.) the total cost of the pencils and pens purchased this year will be three times the total cost of the pencils and pens purchased this year.

Another example of the formulating stage is a released item from PARCC this is a two part item that aligns to the PARCC evidence statement HS.D.3-3a reasoned estimates which states to use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity. A university organizes an event in which all tenth-grade students from three school districts are invited to visit the university campus. Each school district serves students from kindergarten through twelfth grade. The organizers expect 10% of the invited students to attend. Part A, write an equation that can be used to predict the total number of tenth-grade students expected to attend. Describe the assumptions you used to determine your equation. Enter your answer and your work in the space provided.

In Part B the table shows information about districts from which students will be invited and details the number of students in each county. The question asks students to estimate the total number of tenth-grade students expected to visit the university.

A second step in the modeling process is compute stage. At this stage of the process, the students perform mathematical procedures using the model they formulated. Getting a mathematical solution "may involve pencil-and-paper calculations, evaluating a function, running simulations, or solving an equation" (Bliss et al., 2014, p. 7).

In the problem shown students are asked to combine the computing and formulating stages to fully answer the item. It is aligned to G.M.6-2 Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in involving right triangles in an applied setting. According to the Occupational Safety and Health Administration, a ladder should be positioned at an angle such that the distance from the bottom of the ladder to the wall it is leaning against is no more than ¼ of the length of the ladder. The image shows the scenario. A 30-foot ladder is positioned so that the bottom of the ladder is ¼ of its length away from the wall. How far up the wall will it reach? Round to the nearest foot.

A third step in the modeling process is the interpret stage. In this stage the students relate the results of the *Compute stage* to the real-world situation. Thus, they obtain outcomes interpreted within the original problem context.In the problem shown students are implementing the interpret stage that is aligned to A1.M.4 interpret the solution to a real-world problem. An object is dropped off the roof of a building. The function $h(x) = -16x^2 + 64$ models the height, $h(t)$, in feet of an object above the ground x seconds after it is dropped. What is the best description of the meaning $h(2) = 0$ in terms of the context?

a. The object is dropped from an initial height of 2 feet.

b. The object hits the ground 2 seconds after it is dropped.

c. The object travels at a speed of 2 feet per second after it is dropped.

d. The speed of the object increases at a rate of 2 feet per second.

Another example of the interpret stage aligned to A1.M.5, compare the results of a model with real world data. The table shows the distance, in miles, that an athlete will run each week for 6 weeks to prepare for a half marathon. A table titled distance to run each week is shown. The equation $y = 3.25x + 8.5$ models the distance, y, in miles, that the athlete will run in week x. For which weeks is the number of miles given by the equation greater than the actual number of miles the athlete will run? And then the answers choices are weeks 1 through week 6.

Let's look at an item that combines formulating, computing and interpreting. This is aligned to A1.M.6 Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in the standards. A family has some money saved for a vacation they have planned. The vacation will cost a total of $1600. They will add an additional $150/week to the money they have already saved until they have saved enough to pay for the vacation. They will have enough money to pay for the vacation in 8 weeks. If the family was able to save $200/week instead of $150/week, how many fewer weeks would it take them to save enough money for the vacation?

Finally the validating stage of the modeling cycle. In the validate stage the students then test the quality of their model and process. They "reflect on whether the mathematical answer makes sense in terms of the original situation" (Felton et al., 2015, p. 344). For example, they should identify and eliminate mathematical answers, such as negative lengths, that do not make sense in the context of the problem. In addition, the students consider how the outcomes would be influenced if they changed their assumptions or variables (Bliss et al., 2014). After validation, ***the students decide whether to reformulate their model or report their findings***. In our experience and other research, we have found that validation can occur at any stage of the modeling process (Alhammouri, 2016).

Lets look at an items that showcases the validate stage aligned to A1.M.1 choose between competing mathematical models to solve real-world problems. For each one-year period after a car was purchased, its value at the end of the year was 15% less than its value at the beginning of the year. Part A, State whether the value of the car as a function of time after it was purchased is best modeled with a linear function, a quadratic function, or an exponential function, and explain why. Part B, If the value of the car 2 years after it was purchased is $17,918, what was the value of the car when it was purchased? Show your work or explain your answer.

So what distinguishes a content vs modeling item? How to make sure an item is actually measuring mathematical Modeling: Be sure the item aligns to a modeling evidence statement. The item has an appropriate Aspect of Rigor and Cognitive Complexity.Aspect of Rigor for modeling items is generally Conceptual or Application, **not** Procedural. Cognitive Complexity for modeling items is generally level 2 or 3, **rarely** level 1. Modeling items ideally will have layers, meaning that the items require student to connect multiple cognitive dots to determine a response.

So now that you have some insights into what mathematical modeling is and how it is assessed on the MCAP assessment, what can you do? We encourage you to research, read and experiment. One text that is recommended is "Mathematical Mindsets" by Jo Boaler.

"Mistakes, struggles, creativity, beauty, flexibility, equity—Jo Boaler uses these words to describe a vision of mathematics where every student thrives and becomes a mathematical thinker. By following Boaler's roadmap, perhaps we can once and for all lay to rest decades of archaic and destructive notions about what it takes to be good at math."
—Cathy Seeley, Past President, National Council of Teachers of Mathematics and author of *Faster Isn't Smarter* and *Smarter Than We Think*

On behalf of the mathematics team at MSDE and myself, we want to thank you for carving out a little bit of time to learn more about mathematical Reasoning. There are other presentations available as well, including modeling with mathematics and presentations of our holistic rubrics. Thank you!