Transcript for MSDE - Modeling with Mathematics in the Middle Grades.

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Welcome to the Maryland State Department of Education's Office of Mathematics Professional Development Fall 2020 recorded series. This recorded session will focus on Modeling with Mathematics in the Middle Grades.

I would like to begin this session by asking you a question. What is "modeling with mathematics"? Please pause the video and take a moment to think about what your definition of "modeling with mathematics" is.

Now that you have your definition of "modeling with mathematics", here's a definition from the Society for Industrial and Applied Mathematics: "Modeling is a process that uses math to represent, analyze, make predictions, or otherwise provide insight into real world phenomena."

Let me ask you another question: What is the difference between "a mathematical model" and "mathematical modeling"? Pause the video for a moment and think about your definition for each of these phrases.

A mathematical model is just that... it's a model to represent the mathematics. It could be a graph. It could be a number line, an area model, a tape diagram, a geometric shape...

Mathematical modeling is a cycle. It is a cycle used in the mathematical problem solving process. Shown here is a mathematical modeling cycle proposed by the GAIMME report. GAIMME is the Guidelines for Assessment and Instruction in Mathematical Modeling Education. The GAIMME report diagrams the mathematical modeling cycle as follows. A student begins by identifying and specifying the problem to be solved. Then, the student must make assumptions and define variables essential to solving the problem. Following this, a student does the math. This may involve creating a model, performing calculations, and arriving at a solution. The student analyzes and assesses the model and the solution to be sure that they arrived at a precise, accurate, sound solution. The student may need to refine and extend their model in order to produce a more precise or more accurate solution. But once that model and calculations are implemented, and a solid solution is formed, the student reports out the results.

As the GAIMME report shows in this diagram, mathematical modeling is not a sequential, circular process, but rather a web. A student may need to jump back and forth along the modeling cycle to redefine variables, refine or redesign their model, redo their calculations, etc., until an accurate solution is achieved. Shown on this slide is a summary image provided by Dan Meyer. He says that the mathematical modeling cycle is a messy, open-ended process with which a student uses to tackle a genuine, real world problem.

College and Career Ready Standards uses this definition to describe mathematical modeling: "It's a process in which a student chooses and produces appropriate mathematics to model quantities and mathematical relationships in order to analyze solutions, make predictions, solve multi-step problems, and draw conclusions." They use this diagram: the student is presented with the problem, they formulate their solution path, they make their computations they interpret their results, they validate their solutions, and they report out their solutions. But, just as with the GAIMME report, this math modeling process is a cycle, in which the students may need to revisit their solution paths, revisit their models, revisit their computations...until their solution is precise, accurate, and make sense.

Modeling situations and items require students to solve multi-step contextual word problems with a degree of difficulty appropriate for the grade level, requiring an application of knowledge and skills articulated in all of the content domains for that course.

At the end of each course in the middle grades, a student will be taking their MCAP mathematics assessment. (MCAP meaning Maryland Comprehensive Assessment Program.) Once the student takes their MCAP Assessment for Grade 6, Grade 7 and Grade 8, they receive a report. The report speaks to our master claim, which is that a student is college and career ready or on track for being college and career ready. The way that we determine if a student is on track for being college and career ready is it's determined by our sub claims. We have our Content Sub-claim, our Reasoning Sub-claim, and our Modeling Sub-claim. And our Modeling Sub-claim says that a student can solve real-world problems with the degree of difficulty appropriate to the course.

Since Modeling is one of our Sub-claims, we have Evidence Statements to support this Sub-claim. Each grade level in grades 6, 7, and 8 has a set of Modeling Evidence Statements, which can be found in our Evidence Statement documents. The modeling evidence statements begin with a course code, which will be either six for sixth grade, seven for seventh grade, eight for eighth grade. Then the code has an "M" for modeling, and then a number which will correspond to a part of the modeling cycle.

In the middle school grades (Grades Six, Seven and Eight), there is only one modeling standard. It assesses the full modeling cycle. So, it will be designated as a 6.M.1, a 7.M.1, or an 8.M.1, and the modeling evidence statement says: "Choose and produce appropriate mathematics to model quantities and mathematical relationships in order to analyze situations, make predictions, solve multi-step problems, and draw conclusions."

So, the M.1 evidence statement expects students to be able to complete that full modeling cycle. For Grades Six, Seven, and Eight, there is only the one modeling standard. However, there are five modeling sub-standards! And each of these sub-standards is going to assess part of that modeling cycle.

For example, 7.M.1a will assess a student's ability to make necessary assumptions and identify important information to solve the problem. So, there are five steps to the modeling process; there are five sub-standards in our evidence statements. We have 7.M.1a, 7.M.1b, 7.M.1c, 7.M.1d, and 7.M.1e. Let's look at each one of these evidence statements.

First we have our overall evidence statement. The M.1 (whether this would be 6.M.1, 7.M.1, or 8.M.1). It requires the student to be able to follow through with that full modeling cycle; to choose and produce appropriate mathematics to model the quantities and mathematical relationships in order to analyze situations, make predictions, solve multi-step problems, and draw conclusions. You can see pictured here is that full modeling cycle.

7.M.1a, the first sub-standard, focuses on the given problem, and a student's ability to identify the problem that needs to be solved, make the necessary assumptions, and identify important information.

The second substandard, M.1b, focuses on the formulating part of the modeling cycle, which is: Given a real world situation, a student can formulate a mathematical representation of the problem.

The next stage in the modeling cycle, M.1c, the computation step: Given a real world situation, use the mathematical models to compute and draw conclusions. At this stage of the process, the students perform mathematical procedures using the model that they formulated. Getting a mathematical solution may involve pencil and paper calculations, evaluating a function, running simulations or solving an equation.

The next evidence statement, M.1d, is the interpretation part of the modeling cycle, where a student is given a real world situation, can they interpret a solution and what that solution means in the context of the situation.

And the last step of the modeling cycle, and the last evidence statement, is M1e, the validation step: Given a real-world situation, can a student evaluate and/or validate a partial or complete solution.

It is important that we provide opportunities for our students to engage in this modeling cycle and be able to follow each step along the cycle, as well as complete the entire cycle. It's important for us to know, as teachers, how to ask modeling questions that provide those opportunities for the modeling cycle. We need to be able to transform a question from just a question that's asking a student to make a model into a question that actually says what is mathematical modeling. We have to be able to ask a student a question that's going to draw out their ability to model with mathematics. This is more than just asking a student to create a table, or write an equation or show their work.

We see this importance in the Math Practice One: making sense of problems and persevering and solving them. This Math Practice speaks to that entire modeling cycle and how students who can complete that mathematical modeling cycle are mathematically proficient students. Let's take a look at this mathematical practice sentence by sentence to see how this math practice supports that math modeling cycle in our proficient students.

"Mathematically proficient students start by explaining to themselves the meaning of the problem and looking for entry points into its solution." This really is our evidence statement M.1a, when a student is presented with the problem, can they make those appropriate assumptions.

"Students can make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping to a solution attempt." This is our evidence statement M.1b, where a student is able to start to form that solution pathway.

"They consider analogous problems and try special cases and simpler forms of original problems in order to gain insight into the solution." This is where the student begins to make their computations along that modeling cycle.

"They monitor and evaluate their progress and they change course as necessary." So, this is when the student begins to interpret their solution along that math modeling cycle.

And finally, "A mathematically proficient student should be able to check their answers to the problems using different methods and ask themselves, 'Does my answer make sense?'" This is that validation step and our evidence statement M.1e.

Being able to provide opportunities for a student to engage in Math Practice 1 is critical in our classrooms. We need to be able to ask the right questions to develop that mathematical thinking in our students and engage in Math Practice 1, which is to make sense of problems and perseverance solving them.

The chart I have up on the screen is from the Institute for Advanced Study and Park City Mathematics. It was further modified by Melissa Hancock. I want to draw your attention to the top right hand corner of this chart. This chart provides questions that will help us to develop this mathematical thinking in our students, especially with respect to Math Practice 1 (making sense of problems and persevering in solving them). Some questions that we could be asking our students would be: "How would you describe the problem in your own words?" "What information is given in the problem?" "Talk me through the steps you're going to use." "Describe what you've already tried. What might you change?" "What other strategies might you try?" "How might you use one of the previous problems to help you begin this problem?"

Other math practices that align to this idea of modeling with mathematics is Mathematics Practice 4, modeling with mathematics. Insidemathematics.org uses this phrase to describe modeling with mathematics: "Teachers who are developing students' capacity to model with mathematics move explicitly between real world scenarios and mathematical representations of those scenarios. A middle childhood teacher might pose a scenario of candy boxes using multiple flavors to help students identify proportions and ratios of flavors and ingredients. An early adolescent teacher might represent a comparison of different DVD rental plans using a table, asking the students whether or not the table helps directly compare the plans or whether elements of the comparison were omitted. A teacher of adolescents and young adults might pose a kite factoring scenario, in which advanced students are asked to determine the conditions for always creating a particular shape of kite given the dimensions of the diagonals and the angle of intersection."

Ms. Sarah Galasso of Achieve the Core offers this quote: "For many, the misconceptions lie in the fact that they are looking to model 'THE' mathematics. They believe that by using manipulatives or showing students how to do something, that they are modeling with mathematics. In reality, they are modeling the mathematics with other tools. This standard for math practice calls for modeling 'WITH' mathematics, meaning that we need to use mathematics to model the world around us...to apply math skills to solve a real-world life scenario that has meaning for students. It is important to remember that modeling with mathematics is a process, not just presenting mathematical representations of a scenario."

Let's take a look at other questions that we can use to develop this math thinking in our students. We could be asking our students: "What are some ways that you could represent these quantities?" "What's an equation or expression that matches your diagram?" "Could you create a different diagram, a number line, a table...?" "What are some other ways to visually represent this situation?"

In order to create these multiple representations of our problem solving and our math thinking, we need students to be able to use tools appropriately and strategically. So, we have Math Practice 5. Insidemathematics.org describes Math Practice 5 as: "Teachers who are developing students capacity to use appropriate tools strategically make clear to students why the use of manipulatives, rulers, compasses, protractors and other tools will aid in their problem solving processes. A middle childhood teacher might have his students select different colored tiles to show repetition in a patterning task. A teacher of adolescence and young adults might have established norms for accessing tools during students group for tinkering processes, allowing students to use paper strips, brass fasteners, and protractors to create and test...whether they're making quadrilateral kite models, etc.”

Jon SanGiovanni in his blog has this quote: "A tool is anything that aids in accomplishing a task. What is unique about Math Practice 5 is the idea that these tools are used strategically. We must also recognize that tools do not produce the understanding, problem solving, or solutions. When something goes wrong, we need to pause and consider. We adjust. We try the tool again. How does our thinking factor into using these tools? What questions do we want our students asking themselves when they are selecting and using these tools?" Mr. SanGiovanni continues by saying, "The use of the tool itself should support the reasoning, rather than just mere procedure. Most importantly, teachers constantly develop that metacognitive process of tool selection and use. Teachers lift up that thinking process behind the tool as well as the procedure for using the tool. Teachers require our students to either predict what their finding might be prior to using the tool, or teachers require a reflection on the results and make if they make sense. Tools are meant to make sense of mathematics and the world around us. They are meant to improve efficiency and support accuracy."

We can see how this appropriate tool use and strategic tool use really fits into that math modeling cycle. We need to be able to make opportunities in our classrooms for students to use these tools strategically and appropriately. So, questions that we could be asking our students to develop this math thinking around their tools is: "What information do we have in the problem?" "What do you know that's not stated directly in the problem?" "What math tools could we use to visualize and represent this solution?" "Would it be helpful to use a graph, a number line, a diagram? Why would it be useful to use those?" "In what situation might it be more informative or helpful to use a different model?"

Another math practice that helps in this modeling cycle is attending to precision. Insidemathematics.org says: "Teachers who are developing a student's capacity to attend to precision focus on clarity and accuracy of process and outcome in problem solving. A middle childhood teacher might engage his students in a number talk in which students use an in-out table and a plotted graph to guess this teacher's number. An early adolescent teacher might distribute cards with different symbol strings to his students asking them to mingle and group and categorize their symbol strings, explaining and defending their groupings. A teacher of adolescent and young adults might continually probe her students to defend whether their requirements for a particular quadrilateral will always be a case, or whether there are some flaws in the group's thinking in which they need to refine and correct."

So, this goes back to that iteration of the math cycling process where a student might need to attend to that precision of their process and their solution and go back and refine their model and refine their solution path.

We need to make sure that we're asking appropriate questions in our classroom that provide opportunities for a student to reflect back on their work and attend to that precision. Some questions that we could be asking our students are: "What mathematical terms apply to this situation?" "How do you know that your solution is reasonable?" "Is there a more efficient strategy?" "What symbols or math notations are important in this problem?" "How could you test your solution to see if the answers make sense to the problem?"

Lastly, let's take a look at Math Practice 7, which is to look for and make use of structure. As I read through scholastic.com's quote, think about how their quote reflects back on the modeling cycle. Scholastic.com says: "Find patterns and repeated reasoning that can help solve more complex problems. For young students, this might be recognizing fact families, inverses or the distributive property. And as students get older, they can break apart problems and numbers into familiar relationships. Help students identify multiple strategies and then select the best one. Repeatedly break apart numbers and problems into different parts. Use what you know is true to solve a new problem. Prove solutions without relying on the algorithm. Show that they have the right answer without just using steps."

Inside mathematics has this quote: "Teachers who are developing a student's capacity to look for and make use of structure help the learners to identify and evaluate efficient strategies for a solution. An early childhood teacher might help students identify why using counting on is a preferred method, or why multiplication or division could be a more preferable method. A middle school childhood teacher might help the students to discern patterns in a function table. A teacher of adolescents might focus on exploring geometric processes through patterns and proof."

We can see through these quotes how looking for and making use of structure speaks to that modeling cycle. Questions that we could be asking our students are: "What observations did you make?" "What patterns did you find? How did you know that this was a pattern?" "Are there some other problems that are similar to this one?" "In what ways does this problem connect to other mathematical processes and concepts?"

Inside Mathematics provides video excerpts of math lessons so you can see these various math practices in action and see how that problem-solving cycle and that modeling with mathematics connects to these math practices. If you would like to see these videos, please visit insidemathematics.org.

I want to go back to our math assessments, our MCAP assessments, and to take a look at how students are going to be questioned on their assessments, so that we can make sure that we are wording and phrasing and presenting our questions in the same way in our classrooms. On those MCAP assessments, a student's modeling questions are either going to be a Type I question or a Type III question. Type I questions are your machine scored items. They're one point. These can be applied to any of the content standards. These are going to be your questions that are your 'multiple choice', 'select all that apply', your 'drag and drop', etc. They will be one point. All modeling questions are calculator active because we want the student focusing on that modeling cycle and that modeling process, so not to get hung up by tedious calculations. The Type III questions are your human scored response questions. These are your constructed response type questions. They are three or four points. And again, they can be applied to any of the content standards. A student will have a calculator. These are those extended response questions, so they're going to really assess the full modeling cycle, where a Type I question will focus in on just one step or two steps of that modeling cycle. The Type III question is going to assess the student's ability to complete that full modeling cycle. A Type I question is only going to take a student about two minutes to solve where Type III is going to be about seven to ten minutes to solve.

Let's first take a look at Type I questions, and what I mean by the Type I question just focusing on a piece of the modeling cycle (versus a Type III question that would expect a student to complete the full modeling cycle).

Here's a sample Type I question. It reads: David and Karen have a goal to read ten thousand pages together by the end of summer. David reads 80 pages every day. Karen reads 25 percent more pages every day than David reads. David and Karen agree that the model 180 d = 10 000 will tell them how many days it will take them to read 10 000 pages together by the end of summer. They invite Rick to read with them to get to their goal faster. Rick reads 35 percent fewer pages a day than Karen. Which equation can be used to find out how many days it will take David, Karen, and Rick to read ten thousand pages together by the end of summer. This is a multiple choice, so Answer choice A 232 d = 10 000, Answer choice B 245 d = 10 000, Answer choice C 288 d = 10 000, Answer choice D 315 d = 10 000. I would like you to pause the video and find our evidence statement documents. Find which evidence statement this question would align to.

Hopefully you paused the video and found that this question aligns to evidence statement 6.M.1b. It's sixth grade content. It's a modeling question. It's focusing on the ability to create the model that represents the situation. So, this is M.1b. Type I questions are going to assess a student's ability to complete one part of that modeling cycle.

Let's take a look at another Type I question. Actually, this is a Type III question. This is an extended response question. ‘These two maps show the same area at two different scales. Columbus is not on Map A. Map B does not have a scale written on it. Riverside and Gladville are 6.8 centimeters apart on Map A. Riverside and Gladville are 3.4 centimeters apart on Map B. Gladville and Columbus are 1.8 centimeters apart on Map B. The maps are shown. Determine the straight line distance, in miles, from Gladville to Columbus.

This would be a 7th grade modeling item and it actually is the full modeling cycle aligning to 7.M.1. What I would like you to do is pause the video and consider how you could rewrite this item so that it would align to one of the sub-standards, whether that be 7.M.1a, 7.M.1b, 7.M.1c, 7.M.1d, and 7.M.1e. How could you rephrase this question to target just one step along that modeling process? Pause the video and complete that activity on your own.

Let's shift the presentation to now look at Type III items. Here is a Type III item. It's a constructed response item. It says: On Saturday, Craig rode his bike 5/8 of a mile. On Sunday, he rode his bike 1/2 of a mile. Craig added 5/8 and 1/2 to find the total distance in miles he rode his bike on the two days. Craig said 5/8 + 1/2 = 6/10 and plotted 6/10 on this number line (and the number line is shown). Bullet One: Explain why Craig's answer is not reasonable. Bullet Two: Find the distance in miles Craig rode his bike on Saturday and Sunday. Bullet Three: Explained how to use the number line to show your answer is correct. Enter your answer and your explanation in the space provided. You may also use a drawing tool to help explain or support your answer.

Again, I would like you to pause the video, revisit our evidence statement documents for Grades 6, 7, and 8, and find the modeling evidence statement that this would align to.

Thank you. This being a Type III item, it would align to M.1. It asks the student to go through the complete modeling cycle. The content is aligned to Grade 6. So this would be a 6.M.1 evidence statement item.

I want to draw your attention to the bottom right hand corner of the slide, where there is the drawing box. The drawing box is a new tool for students to use strategically as they are answering a constructed response item. The drawing box will be available for all modeling constructed response items. Every Type III item will have this drawing box. A student can use the typing response box, but they can also use the drawing box to support their answer. They could draw a number line, they could draw a graph, they could draw a tape diagram, etc., in that drawing box. When the student's response is scored, the drawing box as well as the numeric entry boxes are both considered in the student's response.

We are now scoring students responses holistically, so we do have new rubrics that we are using when we score a student's response on the MCAP assessment. Shown here on the screen are the modeling holistic rubrics. There are separate recorded sessions in this professional development series that focus on the modeling rubrics, so I'm not going to take time now to explain the rubrics. That pre-recorded session will do that for you.

Let's look at one more Type III item, and let's see if you can find the evidence statement that aligns to this Type III item. This item reads: The Chesapeake Bay Bridge-Tunnel is a 17.6 mile bridge and tunnel crossing at the mouth of the Chesapeake Bay just outside of Virginia. The xy - coordinate plane shown is a map of the Chesapeake Bay Bridge-Tunnel. A car traveling southeast enters the tunnel portion at point B at the same time another car traveling northwest enters the tunnel portion at point A. Assuming the cars travel at the same speed, at what point will the cars pass each other? Use mathematics to explain how you determined your answer.

Enter your answer and your work or explanation in the space provided. You may also use the drawing tool to help explain or support your answer.

Please pause the video and find the evidence statement that this aligns to.

Hopefully you found that this aligns to 8.M.1. It's M.1 because it is the full modeling cycle. It's a Type III item. It is Grade 8 because it assesses systems of equations.

Lastly, I want to point out that because we are moving towards the holistic rubric, modeling items will seldom be broken up into multiple parts, as they used to be done in the old PARCC assessments. The MCAP assessments for our constructed response items will seldom have multiple parts. We want to avoid this scaffolding of 'Part A' and 'Part B' because we want students to be able to approach that modeling cycle in a more open-ended, holistic nature.

I want to summarize this presentation by saying that Mathematical Modeling is a process of choosing and producing appropriate mathematics to analyze those empirical situations to understand them better and improve on decisions. Opportunities to engage in Mathematical Modeling deepens the rigor, relevance, and coherence in a student's mathematical journey.

I sincerely thank you for participating in this recorded series. If you have any questions, please do not hesitate to reach out to me. My name is Nina Riggs. I am the Secondary Mathematics Specialist for the Maryland State Department of Education. My phone number is 410-767-0509. But at the time of this recording, it might be much easier to reach me by email (because of the working from home situation at the time of this recording), so my email is nina.riggs@maryland.gov

Again, I thank you and I encourage you to watch the remaining videos in this recorded series! Thank you!