MCAP Mathematics, Grades 3-5, Assessing Modeling With Mathematics
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Hello and Welcome to the MSDE webinar on MCAP mathematics for grades three through five. Our topic today is assessing modeling with mathematics. My name is Linda Schoenbrodt and I am the elementary mathematics specialist at MSDE.

In our session today, we will focus on these outcomes: To deepen our understanding about Mathematical modeling. What is it and why do we assess it? To deepen our understanding about how modeling will be assessed on MCAP. To develop an understanding of the MCAP evidence statements for modeling. And to become familiar with some instructional tips for implementing modeling practices in the classroom.

What do we mean by modeling in Mathematics? Let us begin by defining it.

Modeling is one of the standards of mathematical practice. SMP 4 says students need to model with mathematics. They use mathematics to model the world around us. Let us look at how SMP 4 defines modeling with mathematics.

Students who model with mathematics are able to apply and use the mathematics they know to solve problems in everyday life and provide a solution with mathematical representations. They reason quantitatively and abstractly. They are able to decontextualize and contextualize the context of a problem and its solution. They're able to interpret and describe a situation mathematically. And reflect on whether the results make sense, possibly improving and revising the solution. They also ask, How can I represent this problem mathematically?
In our elementary grades, there is some misunderstandings about what modeling means. This misconception is really based on the vocabulary we use in the elementary grades about models.

Modeling is often thought of as the concrete and pictorial models we use to help students understand the mathematics. When we use manipulatives and diagrams or pictures we are representing the mathematics. For many of us, when we hear the word models or modeling, we may think of the CRA process that is used to develop conceptual understanding.
CRA stands for concrete, representational and abstract. As students move through these phases they are learning a mathematical concept and they're also learning that a concept can be represented in a variety of ways. However, this is not the definition of modeling that the SMP 4 is explaining. The word model in this case causes some confusion. Modeling with mathematics is really based on a problem-solving process that includes the use of representing the mathematics in the problem. It may be less confusing if we would use the word representations when we are referring to the concrete materials or diagrams that we use to represent the math. Let us take a look at a few examples to help us better understand the differences between representing the math and modeling.

Here is our first example, Use an array to model the following multiplication problem five times seven equals. Take a few minutes and think about this problem. What do students need to do to solve it? This example asks students to create a representation of 5 times 7 and it specifically states that they should use the array model An example from SMP 4 would ask students to apply mathematical concepts and skills to solve a real life scenario using a strategy that is meaningful to them. The solution would reflect the context of the problem, includes the correct interpretation of the quantities, and is mathematically correct and accurate.

Here is another problem. George wants to buy a new bike. He has thirty dollars saved but needs one hundred sixty dollars to buy the bike. George needs your help creating a savings plan so that he will have enough money in five weeks to buy his bike. Explain the savings plan that will help George buy his bike. Compare this problem to the one that we just saw. How are they different? This problem has a context that would be familiar to students. In order to solve this problem, students need to go through several steps to determine the solution. They need to first decontextualize the problem in order to understand the situation and think about it mathematically .They need to understand the meaning of the quantities. They need to identify which operations will be used to solve the problem. Students need to use all of this Information to provide a solution that is an accurate and precise mathematical representation. This problem also allows students multiple entry points and different ways to represent the problem. There are several strategies students could use to solve this problem, and different ways to explain their plan for George to buy his bike.

When we compare the two problems, we can see that the array problem only requires students to represent 5 times 7with a pictorial representation using an array. The second problem requires students to model with mathematics. Keep George's problem in mind as we look at the modeling cycle.

On this slide, you will see the modeling cycle. The modeling process is made up of these components. Component one- Identify the problems to be solved. In this phase, students identify something in the real world we want to know, do, or understand. The result is a question in the real world that needs to be solved. Students begin by identifying the problem from the context of the item. The second component-Make Assumptions and Define Essential Variables. Students look for and make meaning of any given quantities. They decide what information is needed and not needed to solve the problem. If there is missing information, students will need to make assumptions and use their knowledge of mathematics to provide needed information. Component 3- Do the Math and Create a Solution. Students decontextualize the situation into mathematical terms and translate the real world problem into a form of a mathematical representation. This requires a thorough understanding of the problem. The solution represents the context of the problem along with the quantities found in the problem. The solution to the problem is mathematically accurate and precise. Component 4- Analyze and Assess the Solution. In this phase, students contextualize the mathematical representation. The students may consider, Does the solution address the problem? Does the solution make sense when we translate it back into the real world? Are the results practical and the answer is reasonable? Keep in mind that this process is very fluid. Students may move back and forth between the different phases before they determine how to best represent the problem mathematically.

Now that we have a better understanding of what is meant by modeling with mathematics, let us learn how modeling will be assessed on the MCAP mathematics assessment, for grades three through five.

There are two different types of items for modeling on the MCAP assessment. The first modeling item we will discuss is the type one items. This item is machine scored and is worth one point. This is a different item from PARCC. The PARCC items did not have one point modeling items. The type one items are made up of two evidence statements. One evidence statement will be one of the five modeling evidence statements. The second evidence statement will be one of the content evidence statements that are aligned to the grade's Maryland College and Career Ready Standards.

The second type of modeling item is a Type 3 item. Type 3 items are constructed response items, which are humans scored. This is the type of modeling items we saw on the PARCC assessment. In grades 3 and 4, they will receive three-point constructed response items and grade five will receive three and four point constructed response items. The type three modeling item is also made up of two evidence statements. One modeling evidence statement and one content evidence statement based on the Maryland College and Career Ready Standards for the grade. Since these items are answered with students written responses, they will be hand scored by a committee of Maryland educators using a holistic rubric.

This slide shows a copy of the MCAP holistic rubric for three and four point constructed response items. You can find a copy of these rubrics on the same web page where you found the link to this webinar. You may want to consider using this rubric to score your students' work. In addition, MSDE is offering a webinar on how to use the modeling holistic rubric. It can also be found on the same web page as this webinar.

Another new feature of the MCAP Mathematics Assessment is that all students in grades three through five will have access to an online calculator. The calculator will be available for only reasoning and modeling items. There are two versions of the online calculator. Students will be able to select the online calculator that they would like to use. Both models are displayed on the screen. Both calculators are five function calculators. A five-function calculator has four basic operations, addition, subtraction, multiplication, and division, and a percent and square root key. You can help students prepare for using the calculator during the assessment by providing them with an online calculator to use during instruction. Desmos offers free access to their five-function calculator. Desmos also offers a braille version of the calculator.
The links on the screen provide direct access to both calculators.

Here are a few tips for calculator use. Students should first read the problem and then they should solve the problem but record their answers and their work and solution on a separate sheet of paper. After they have all their work completed, they use the calculator to do the computations. Then once they have a solution on their calculator they should check the answer on the calculator to determine if the solution is reasonable and correct. Their next step is to transfer the work and solutions to the online assessment format.

In this section, we will learn about the MCAP modeling evidence statements for grades three through five. We will also view a sample item for each evidence statement. An evidence statement describes the knowledge and skills that an assessment item elicits from the students. The MCAP evidence statements are derived directly from the Maryland College and Career Ready Standards. The examples that you will see are based on the context of a problem that was a release PARCC item. The same item context is being used for all five of the evidence statements so that you can see how the modeling process evolves. As students move through the processes and the phases of the modeling cycle the MCAP assessment will never use the same context on all of the different evidence statements. Please keep in mind that example items should not be considered a template for all modeling items. The meaning (intent) is for them to clarify the meaning of the evidence statements.

The MCAP elementary modeling evidence statements are aligned to this modeling cycle. Let us take a look at the evidence statements to see how the modeling evidence statements are aligned to this modeling cycle.

Here are the five evidence statements for grades three through five. All three grades will have the same wording for the evidence statements. The difference in the items will be the grade level content standard that is paired with the modeling evidence statement.
Let us read through these. M1-1 Determine the problem that needs to be solved in a real world situation. M 1-2 Determine the information that is needed to solve a problem in a real-world situation. M.1-3 Identify the mathematics that is needed to create a solution path for a real-world situation and M. 1-4 Create a solution path that represents the mathematics needed to solve a real world situation. M 1-5 To evaluate a partial or complete solution to a real-world situation.

The modeling evidence statements use a different value or scoring for these items. The evidence statements for M.1-1 M.1-2 and M1-3 will only be worth one point .These are machine scored items only. There will not be any constructed response items.
Evidence statements M.1-4 and M1-5 will all be type 3, constructed response items only. There will not be any one-point items for these two evidence statements. Let us take a look at each of the evidence statements individually.

Our first evidence statements M 1-1 is aligned to the first phase of the modeling cycle, Identify the problem to be solved. M.1-1 students will focus on just phase one of the modeling process, to identify the problem to be solved. In phase, one of the modeling process students read the problem and decontextualize the problem to make sense of it. Students are asked to determine the question that could be asked to solve the problem. Let us take a look at this problem. Two classes are working on a science experiment. Each class has 23 students. The students will work with a partner at a table. Which question could be asked, using the information given in the problem. There are four answer choices. And they are, A. How many students enjoy the science class? B. How many science books are needed for one class? C. How many students can be seated at a table? D. How many tables are needed for all the students from both classes? Notice that students are not required to solve the problem; they only need to select the question that could be asked to solve it.

Here are a few instructional tips for helping students better understand the modeling evidence statement M. 1-1. Provide a problem without a question or answer choices then have the students decide what question could be asked to solve the problem? A second suggestion would be to provide a problem, ask the students to discuss the problem, but then have the students explain the problem in their own words and then think of a question that could be solved, based on their information in the problem. And a third suggestion is to, Use numberless word problems and ask students to think of a question that could be solved with the information given. All of these suggestions only require students to practice the first phase of the modeling cycle, to identify the problem to be solved. By identifying the question that could be asked to solve the problems, students are beginning to decontextualize the context of the problem.

Here is our second evidence statement for modeling M.1-2. Determine the information needed to solve the problem and it is aligned to the second phase of the modeling process-To make assumptions and define essential variables. In this, phase students begin to analyze the information in the problem. They decontextualize the problem to make sense of the quantities that are given in the problem. They decide what information in the problem makes sense and are needed or not needed to solve the problem. They may decide that some information is missing and will need to make assumptions in order to solve the problem. Let us take a look at this problem. Two classes are working on a science experiment. Each class has 23 students. The students will work at tables that can seat four students. How many tables are needed for all the students from both classes? What three pieces of information are needed to solve the problem? There are six answer choices. When you read the question, you will notice that the word three is bolded; that is because this is considered a multiple select item. In a multiple select items, students need to select more than one correct answer. In the elementary grades, we will always identify the number of correct answers the students need to provide. Let us think about this. Remember that M.1-2 items are machine scored and they are one point. Students will select or identify their answer. They do not need to write a solution path.

Here are a few tips to help students solve problems for M.1-2.Consider giving students a problem and ask them to read it. Have them list the information from the problem that is needed to answer the problem. They do not need to do a solution. Just help them practice identifying the information that is needed. Another suggestion is to provide a problem and ask the students not to solve it. Ask them to identify the quantities that are in this problem and identify what they mean. Or you could ask students to label the quantities. And a third suggestion would be to provide a problem to the students and ask them to draw a representation of the problem; and then label the information needed to solve the problem.

Here is our third modeling evidence statement M.1-3. It is aligned to the third phase of the modeling cycle. M1-3 items are one-point items. Students are not asked to write the solution. This evidence statement is kind of a mini step for students' thinking of the operations before they write an entire solution. So let us read this item. Two science classes are working on a science experiment. Each class has 23 students. The tables in the science lab can each seat up to four students. How many tables are needed for all the students from both classes?

The question is, Which operations will be used to solve the problem? A. only division B. only multiplication C. First division and then addition. D. First addition and then division. For the M1-3 evidence statements, students need to use all the information from the problem and identify the mathematics that is needed to solve the problem and or the sequence of the operations.

Here are some suggestions for helping students get a better understanding of M1-3. Again, we are not going to ask students to write a whole solution path but consider providing students with a problem and having them read it, but ask them to first identify the mathematics that they need to solve the problem. Or you could have students list the operations that they will use in the sequence that they will need them. The other thing you could do is have students then go back to the problem and see if they can justify their solutions by looking at the problem that understands why they selected the operations that they did.

Our fourth evidence statement M.1-4 is aligned to the third phase of the modeling cycle. These are the kinds of problems, story problems/ word problems the kids are used to solving. These are constructive response items. In this evidence statement, the items will ask students to solve the problem and represent the solution using mathematical representations. Students will need to provide answers based on what the item is asking for. Their solution should be precise and mathematically correct. Students will have available to them a drawing tool along with the equation editor to provide their answers. Let us read this problem together. Two science classes are working on a science experiment. Each class has 23 students. The tables in the science lab can each seat up to four students. How many tables are needed for all the students from both classes? Then there are two bullets. Bullet one, Write an equation to find the least number of tables needed for all of the students from both classes in the science lab. Bullet 2-Use a letter for the unknown value in your equation. The direction line says to; Enter your answer and your work or explanation in the space provided. (Which would be the equation editor). You may also use the drawing tool to help explain or support your answer.

It is very important for students to very carefully look at these problems and make sure that they follow the directions of what the problem needs to include. Think about this problem; think about all the things that we have said earlier about good modeling problems. We want to provide many experiences (for students) to apply the phases of the modeling cycle to solve these problems. We want to make sure the problems that we provide for students allow them to: apply and use the mathematics they know to solve problems in everyday life and provide a solution with mathematical representations. They need to be able to reason quantitatively and abstractly. They need to be able to decontextualize and contextualize as part of the modeling process. Students need to use mathematical representations to describe the situation and interpret the mathematical situation. It should allow them to reflect on whether the results make sense, and possibly allow them to improve or revisit the model.

Here is an example of modeling M.1-5. These items only focus on the last step in the modeling process-to analyze and assess the model. For this item, students need to contextualize the solution. Which means they need to look at the solution, look back at the problem, to ensure that the solution correctly represents the problem. If the answer is correct, students need to explain how the model represents or reflects the context of the problem. They need to justify the solutions, the quantities and what they mean, why the operation is correct and interpret remainders if appropriate. If the solution does not correctly represent the problem, students need to explain why it is not correct based on the information from the problem. Let us take a look at what this problem says. Two science classes are working on a science experiment. Each class has 23 students. The tables in the science lab can each seat up to four students. How many tables are needed for all the students from both classes? One student's work and explanation for the problem is given. And they list the steps 23 times 2 equals 46 46 divided by 4 equals t, t equals 11 with a remainder of 2 .The total number of tables however the final answer is 12 tables. Because of the remainder, the students had to interpret the remainder to go up to a second/another table. Notice that the M.1-5 item is three points. It is constructed response. The items provide a solution or sometimes students will be asked to give their own solution.

Here is some instructional tips for M.1-5. Instead of giving students the whole problem, you can give them the problem with the solution, and have them evaluate the solution and explain how the student work correctly or incorrectly represents the problem. You really want them to go back to the problem and draw out. What information in the problem that tells you that the work is correct or incorrect? The second suggestion would be to give students a problem and have them solve it, but then have them exchange their work with another student to analyze the solution; and that other student can explain how the solution correctly or incorrectly represents the problem. It is very important that you hold students accountable for using precise and correct mathematical representations.

This concludes the information that we wanted to share with you on modeling. So let us go back to our session outcomes. There were three big ideas that we wanted to cover today. One was to define mathematical modeling. We took a lot of time looking at what mathematical modeling means. We understand now that it is not just the representations

However, an actual process is used to solve problems. Our second outcome was to explain how modeling will be assessed on the MCAP and for this (outcome) we showed you the evidence statements for the modeling items and we've also talked about the different features that will be available on the MCAP for students, to help them as they are solving the problems. Then our third outcome was to provide you with some instructional tips for modeling. We can see how modeling is not just something for assessment but rather it is a process the students need to use when they are solving problems every day.

In closing, thank you so much for being with us today. I hope you have a deeper understanding of what modeling with mathematics means. I really want to thank you so much for all you are doing to make your students mathematical thinkers. If you have any questions, feel free to contact me at the provided email (address) linda.schoenbrodt@maryland.gov.
Thanks for joining us!