# **Transcript for** MSDE - MCAP Mathematical Reasoning in High School

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Hello and welcome to the Maryland State Department's Presentation of Reasoning in High School Mathematics. My name is Michael Turlik and I am a Mathematics Assessment specialist at MSDE.

The outcomes for this presentation are as followed: Investigate what is meant by reasoning in high school mathematics. Understand how reasoning is assessed on the MCAP mathematics assessment. This presentation is to be shared with teachers, parents and other stakeholders in a student's mathematical education. Our ultimate aim is to promote reasoning as integral part of mathematics instruction.

So before we jump in to how mathematical reasoning can be ingrained into classrooms and assessments we have to discuss what reasoning actually is. A great resource from NCTM called Focus in High School Mathematics: Reasoning and Sense Making has this to say about mathematical Reasoning: "In mathematics, reasoning involves drawing logical conclusions based on evidence or stated assumptions." Reasoning at its core is understanding how students think about mathematics.

A great image that depicts the mindset encouraged in students as they approach reasoning within mathematics is shown here. The student in the image is reading, for our sake let's say a word problem and is reasoning about it, what are they seeing, what are they noticing, what are they thinking, feeling and wondering? These are some of the questions that students and teachers should be asking daily to get student's to think about mathematics.

Again from NCTM's 'Focus in High School Mathematics: Reasoning and Sense Making' is the following: It is not enough for students to know only how to perform mathematical procedures or recall facts. Gone are the days of the timed multiplication quizzes or the worksheets with dozens of factoring questions. Those resources do not provide students with the ability to reason with mathematics and show their thinking. Students must learn to reason and make sense of mathematics so that they are able to use math in meaningful ways. In today's world, students need to develop critical thinking skills to succeed in mathematics and in life.

Why is reasoning important? If students truly understand what they are learning, they are more likely to retain the concepts and skills that they are taught. At a high level a focus on reasoning allows us to organize our curricular materials around central ideas or themes developed through reasoning and sense making introduces coherence that may ultimately allow a streamlining of curriculum.

Reasoning with mathematics is hard to capture in text, it is really experienced when talking with student's and in a classroom environment but there are habits that students and educators can use to develop reasoning mindsets in classrooms. These are small things that can be incorporated into classroom instruction to promote reasoning and should be done vocally and in large and small groups while students are approaching a problem. The first is to analyze the problem. Reading through the problem with a group of student's, having them pick out what information is important, what information is superfluous and what the question is asking are all ways to analyze a problem with a reasoning mindset. Implementing a strategy, how are you going to go about solving the problem? Seeking and using connections, what skills do you already know that can help you in this situation? And finally reflecting on a solution, does your answer make sense? Does it fit the context of the problem?

Does it fit what would actually happen in the real world? This lets us get beyond the 'well that is what my calculator said' and allows for a more in depth discussion of mathematics. Many reasoning habits fit in more than one category, and students should move naturally and flexibly among them as they solve problems and think about mathematics.

There are other ways that reasoning can be incorporated into classroom instruction. It is important to also note that reasoning should occur in mathematics instruction every day. It not something that is held for the end of a unit or as a capstone after all content is taught. Easy ways to promote a reasoning mindset could be to ask questions such as "What's going on here?" or "Why do you think that?" are small steps to add a level of reasoning to lessons. Teach the why behind procedures and formulas and connect those procedures and formulas to real world scenarios. Allow students to experiment with what they know before introducing a new formula. Analyze solutions to problems in terms of the context and discuss different solution paths. Discussing different solution paths as a class opens mathematics up to multiple ways to approach and understand a problem and can help students see mathematics as a rich and wonderful subject as opposed to a strict, one way only subject.

So we looked at what reasoning is, why it is important and how to incorporate it into classroom instruction, but how is reasoning assessed? Regardless of our educational aims, students, teachers, administrators, and many others equate **what we test with what we value.** This means that assessments need to include items that call for reasoning and sense making. Such items should, for instance, require students to explain their thinking and reasoning or show that they can use mathematics flexibly in non-routine situations. Students need to experience these kinds of assessments regularly in high school mathematics classrooms, reasoning assessment items should not be held for only high stakes testing but rather should be included as formative and summative assessments in classrooms.

Reasoning with mathematics plays a large role on our state assessment.

As a state, we give assessments to meet federal guidelines for accountability for our students. Our master claim we are trying to assess with our mathematics assessments is if student's are on-track to being college and career ready in mathematics. How we can prove that they are is by having three subclaims that are reported out when students take one of our assessments. The content subclaim asks students to solve problems related to the grade or course level content. The reasoning Subclaim asks students to express grade level appropriate mathematical reasoning and the modeling Subclaim asks students to solve real-world problems with a degree of difficulty appropriate to the course.

If we take a deeper dive into our subcategories at the high school level, you can see how each of the categories are weighted. All of the assessments for the high school level are approximately 48 points, where the content category represents about half of those points and reasoning and modeling both make up a fourth of those points.

Reasoning items on the MCAP assessment are written based off our evidence statements. These evidence statements can be found on our website and exist for each of the three end of course exams. An example of the reasoning section of the evidence statements is shown below. The code for the statement is listed on the left hand side. In A1.R.1 the A1 stands for algebra I, the R for reasoning and the 1 for the order of the evidence statements. G is used for geometry and A2 is used for algebra II. The next column shows the reasoning evidence statement and the last column shows clarifications or limits as to what can be assessed for that evidence statement.

All items that assess reasoning evidence statements also need to be aligned to at least one content evidence statement in the same course. The content evidence statement is considered an "Additional Standard"

On the MCAP high school mathematics assessments, our items are broken up into 3 types, type I items which are 1 point machine scored items, and Type II and III items which represent modeling and reasoning Constructed response items that are 4 points and are human scored. Items that assess students ability to reason with mathematics are found in both type I and Type II items. New for the state of Maryland, is that reasoning is now assessed with 1 point items. This is positive news in two ways, one, it allows students to show their ability to reason with mathematics in a more accessible way as one point items generally take less time to complete. Secondly it allows for reporting on the reasoning Subclaim to be based off of both machine scored and human scored items.

Type I and Type II items may be written for each reasoning evidence statement unless stated in the evidence statement. Type I items are machine scored, are 1 point per item, and can be applied to any of the content standards unless otherwise noted in the evidence statements. All reasoning items are calculator active, meaning that a student will always have a calculator when these items appear on an assessment. Four items from this grouping will appear on each assessment. Type II items are constructed response reasoning items. They are 4 points per item, can be applied to any of the content standards, unless otherwise noted on the evidence statement documents. Like the type I reasoning items, all Type II items are calculator active and two of these items will appear on each high school assessment.

So now we are going to look at what makes an item a reasoning item vs. a content items. We are going to start with a type I item, which again is worth one point and machine scored. For the Content version of the question we have, What are the solutions to the equation $(x-3)^2-4=0$? There are two blanks where students input their answers. This question aligns with the evidence statement A.REI.B.4b, solve quadratic equations with rational number coefficients… If we look below, you can see an interpretation of this content but put in the reasoning lens. Consider the equation $(x-a)^2-b=0$, where a and b are both positive real number. Which statement about the solution or solutions to the equation must be true? A.) the equation has no real solutions, B.) the equation has exactly one real solution and the solution is positive. C.) The equation has exactly two real solutions and both of the solutions are positive or D.) the equation has exactly two real solutions and at least one of the solutions is positive. This also aligns to A.REI.B.4b but also aligns to the reasoning evidence statement of A1.R.1, given an equation reason about the number and/or nature of the solutions. What makes this question more reasoning than content is that it allows students to think more deeply about what they know of quadratics and how the values of a and b help predict the number of real solutions. Students are using their conceptual knowledge as opposed to procedural fluency.

Another example of a content question is below. A paper cup has a base radius of 3 centimeters and a height of 5 centimeters as shown in the image. What is the volume, in cubic centimeters, of the paper cup? Blank cubic centimeters. This aligns to G.GMD.A.3 which says to use volume formulas for cylinders, pyramids, cones and spheres to solve problems. This is a straightforward content question because students have to use the general volume formula and plug and chug to get the correct answer.

Now if we look at the context of this problem again but add some ambiguity, it allows us to see if a student understands the concept of what we are asking as opposed to plug and chug. A cone has a base radius of 3 centimeters and a height of 5 centimeters. A student correctly calculates its volume to be $15\pi$ cubic centimeters. The student thinks that a simpler formula of a cone is $V = \pi rh$ because $\pi(3)(5) = 15\pi$. Which statement explains the conditions for which the student's claim would be true? A.) The claim is true only when the height is 5? B.) the claim is true only when the radius of the cone is 3. C.) The claim is true regardless of the dimensions of the cone. D.) The claim is true whenever the product of the base and height is 15. This still aligns to G.GMD.A.3 but it also aligns to G.R.3, determine cases where a given proposition is true or false. In this item students are asked to prove a claim is true or false. In the claim you can see that someone has already tried to use what they know to reason with mathematics and it is now the students turn to reason with their prediction.

For our last type 1 item we will look at an Algebra II item. For the content item we have students finding the solution to the equation $4 = \sqrt{x+5}$. Students are asked to then put their answer in the blank in the space provided. This aligns to A.REI.A.2 which states to solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. This is a very procedural and straight forward question. To make the item fall more into the reasoning camp we add a reasoning evidence statement, in this case A2.R.1 which states given an equation reason about the number and/or nature of the solutions, and reframe the questions as follows. Consider the equation $a = \sqrt{x+b}$, where a and b represent real numbers. Which statement is true about the number of real solutions to the equation to the equation? A.) the number of real solutions depends only on the value of a. B.) the number of real solutions depends only on the value of b. C.) the number of real solutions depends on the relationship between a and b. Or D.) the value of a and b do not affect the number of real solutions.

So how can we distinguish between items that assesses reasoning vs. an item that assess content? For one if we can check to see if a reasoning evidence statement fits the item. We can check the cognitive complexity and aspect of rigor (Achieve the Core). Reasoning items tend to be set up so that students are making generalizations with mathematics.

We looked at 1-point reasoning items now we are going to look at 4-point reasoning items. Constructed response items are human scored where students need to be given an opportunity to show their work and/or explain their thinking. Some aspects that apply to the MCAP assessment but that you can adopt into your classroom is that items are scored with a 4-point holistic rubric, items should be able to be completed in 10 minutes and parts of the question should be related but not dependent.

This slide shows the holistic rubric we use for our Reasoning CR items. The rubric and a separate recorded webinar can be found on our website.

Let's first look at an example of an algebra I item that aligns with A.REI.C.6, Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. And also with A1.R.8, Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). The system of equations shown is graphed in the xy-plane. Determine the coordinates of the solution of the system algebraically. Then show that your solution is valid. Show your work or explain how you found your answer.

Another example of an algebra I CR item is shown that aligns to both F.BF.B.3, Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of k given the graphs and A1.R.10 Express reasoning about transformations of functions.

The next CR item is a geometry item that aligns to G.GPE.B.7, Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula and G.R.6 Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions.

The last CR item is an algebra II item that aligns to N.CN.C.7, Solve quadratic equations with real coefficients that have complex solutions and A2.R.4 Identify an option that would refute a conjecture/claim.

Now that you have some insight into what mathematical reasoning is and how it is assessed on the MCAP mathematics assessment, what can you do? Research, read and experiment:

Mathematical Mindsets – Jo Boaler

"Mistakes, struggles, creativity, beauty, flexibility, equity—Jo Boaler uses these words to describe a vision of mathematics where every student thrives and becomes a mathematical thinker. By following Boaler's roadmap, perhaps we can once and for all lay to rest decades of archaic and destructive notions about what it takes to be good at math."
**—Cathy Seeley,** Past President, National Council of Teachers of Mathematics and author of *Faster Isn't Smarter* and *Smarter Than We Think*

Routines for Reasoning – Grace Kelemanik

"This book honors the art of teaching and the brilliance of students thinking. It will help teachers think a lot harder about what the mathematical practices mean...This book should be on every mathematics teacher's bookshelf." --**Elham Kazemi, Geda and Phil Condit Professor in Mathematics Education, University of Washington; coauthor of *Intentional Talk***

On behalf of the mathematics team at MSDE and myself, we want to thank you for carving out a little bit of time to learn more about mathematical Reasoning. There are other presentations available as well, including modeling with mathematics and presentations of our holistic rubrics.

Thank you!